

UNIT - I

DRIVE CHARACTERISTICS

Advantages of Electrical Drives:

- * They have flexible control characteristics
- * It can be provided with automatic fault detection system.
- * They are in wide range of torque, speed, and power.
- * They are adaptable in wide range like almost any operating conditions.

Classification of Electrical Drives

→ Group Drive

It consists of a single motor, which drives one or more line shaft supported on bearings. The line shaft may be fitted with either pulleys and belts or gears, by means which a group of machines or mechanisms may be operated. It is also called as shaft drive.

System employed for motion control are called drives.

Drives employing electrical motors are called electrical drives.

Advantages:-

- * single large motor can be used instead of a number of small motors.
- * The rating of single large motor may be approximately reduced.

Disadvantages:

- * No flexibility
- * New machine connected to main shaft is difficult.
- * If some of the machine are not working, the losses are increased.

→ Individual drives

In this drives, each individual machine is driven by a separate motor.

Ex single spindle drilling machines
lathe machine

Disadvantages:

The energy is transmitted to different parts. Hence occurs some power loss.

Multimotor Drive:-

In this system, several motors used, each of which serves to ~~act~~ actuate one of the working parts of the driven mechanism.

Ex: Metal cutting machine tools

Paper making machine

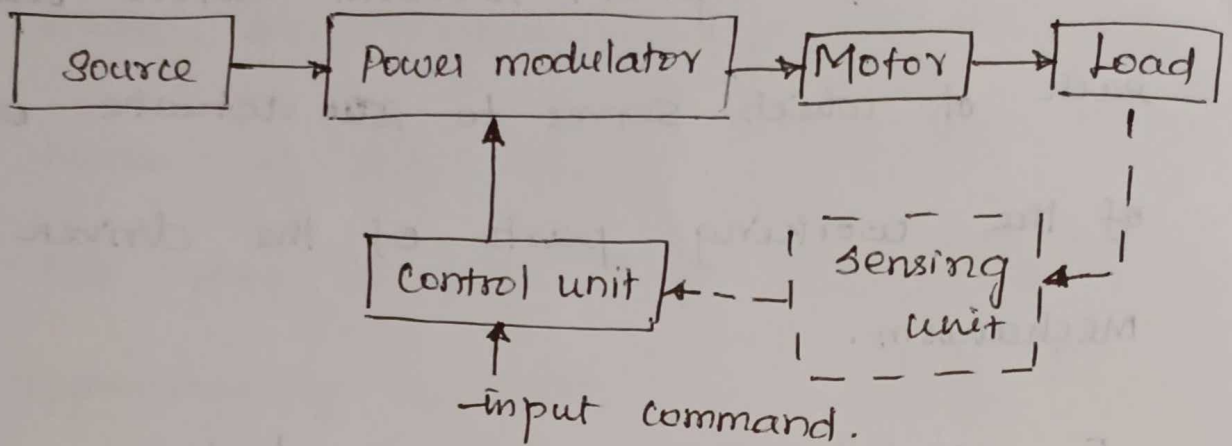
rolling mills.

Crane.

choice or selection of Electrical Drives:-

- * steady state operation requirements
- * Transient operation requirements
- * Requirements related to the source
- * Capital and running cost, maintenance needs.
- * space and weight restrictions.
- * Environment and location
- * Reliability.

General Drive System:-



Classification of Electrical Drives:-

* AC Drives

* DC Drives.

Comparison between AC & DC Drives:-

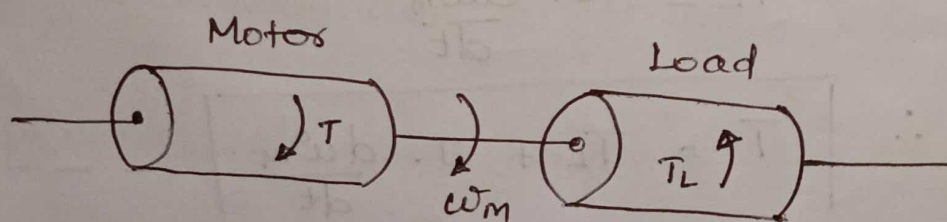
	DC Drives	AC Drives
1.	power & control circuits is simple & expensive	The power & control circuit is complex.
2.	Required frequent maintenance	Less maintenance
3.	speed and design ratings are limited due to commutation	Speed and design ratings are no upper limit.
4.	Fast response and wide speed range smoothly by conventional and	solid state control the speed range is wide but in

	Solid state control.	Conventional method Speed is limited
5.	Power/weight ratio is small.	Power/weight ratio is large.
6.	It is used in some certain locations	It is used in all locations.

Dynamics of Motor Load system:

(i) Fundamentals torque equation:

A motor generally drives a load through some transmission system, while the motor always rotates the load may rotate and may undergo a translational motion.



The motor load system can be derived by the following fundamental torque equation

$$T - T_L = \frac{d}{dt} (J \omega_m) \quad \text{--- --- --- } \textcircled{1}$$

$T \rightarrow$ developed motor torque

$T_L \rightarrow$ load torque

$J \rightarrow$ Polar moment of inertia

$\omega_m \rightarrow$ Angular velocity of motor shaft.

then,

$$T - T_L = J \cdot \frac{d\omega_m}{dt} + \omega_m \cdot \frac{dJ}{dt} \quad \text{--- --- --- (2)}$$

Equation (2) only applicable for

- variable inertia drives
- reel drives
- industrial robot.

For driven with constant inertia

$$\frac{dJ}{dt} = 0, \text{ then}$$

$$T - T_L = J \cdot \frac{d\omega_m}{dt}$$

$$\therefore \boxed{T = T_L + J \cdot \frac{d\omega_m}{dt}} \quad \text{--- --- --- (3)}$$

The torque developed by motor is counter balanced by a load torque ' T_L ' and a dynamic torque $J \left(\frac{d\omega_m}{dt} \right)$. The torque component $J \left(\frac{d\omega_m}{dt} \right)$ is called the dynamic torque because it present only during the transient operation.

classification of loads:-

- * Active load torque
- * Passive load torque.

The load torque which have the potential to drive the motor under equilibrium conditions are called active load torque.

Ex: Torque due to force of gravity

Torque due to tension, compression torsion undergone by an elastic body.

The load torque which always oppose the motion and change their sign on the reversal of motion called Passive load torques..

Ex: Torque due to friction, cutting.

components of load torque:-

* Friction torque:- (T_f).

The friction will be present in the motor ~~torque~~ shaft and also various parts of the load.

* Windage Torque (T_w):-

When a motor runs, the wind generates

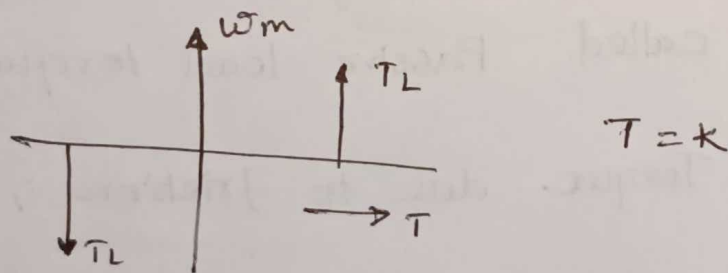
a torque opposing the motion. This is known as winding torque.

* Torque required to do the useful mechanical work

The nature of this torque depends upon the type of load. It may be constant and independent of speed, it may be some functions of speed, it may be time invariant or time variant and its nature may also vary with the change in the load's mode of operation.

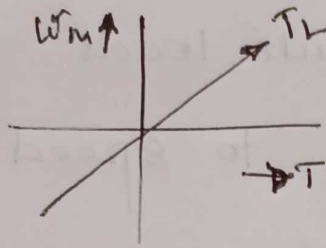
characteristics of different types of Loads:-

* constant torque load:-



Most of the working machines that have mechanical nature of work like shaping, cutting, grinding or shearing, required constant torque irrespective of speed.

* Torque is proportional to speed:-



$$T \propto N$$

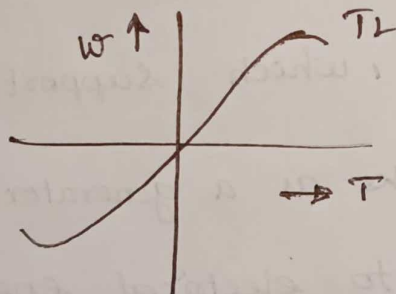
$$T \propto \omega$$

$$T \propto k\omega$$

Separately excited DC generators connected to a constant resistive load, eddy current brakes and calendaring machines have a speed torque characteristics given by

$$T = k\omega$$

* Torque Proportional to square of the speed:-



$$T \propto \omega^2$$

$$T = k\omega^2$$

The load torque is proportional to the square of the speed.

Ex: Fans, rotary pumps, compressors and ship propellers.

* Torque Inversely proportional to speed:-

Certain types of lathes, boring machines, milling machines, steel mill coiler and electric

traction load exhibit hyperbolic speed torque characteristics. In such loads the torque is inversely proportional to speed torque characteristics.

$$T = k/\omega$$

Multi motor operation :-

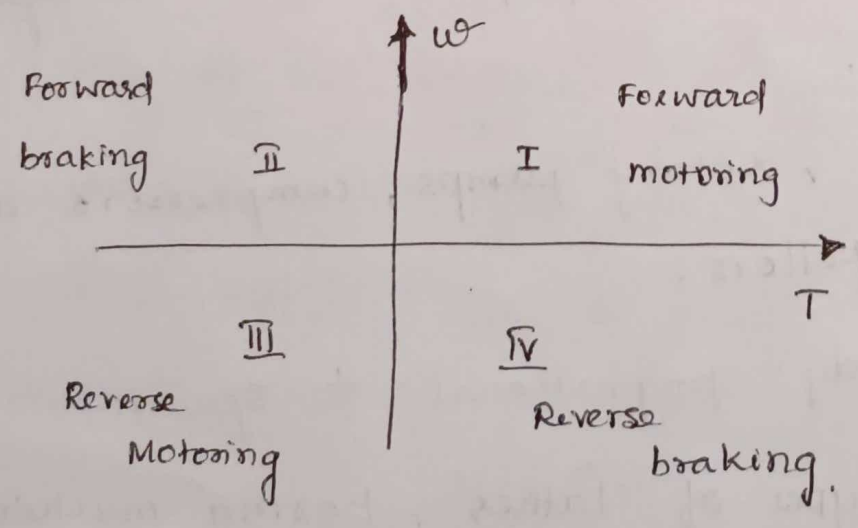
Multi quadrant operation :-

A motor operate in two modes

- Motoring
- braking

In motoring, it converts electrical energy to mechanical energy, which support motion.

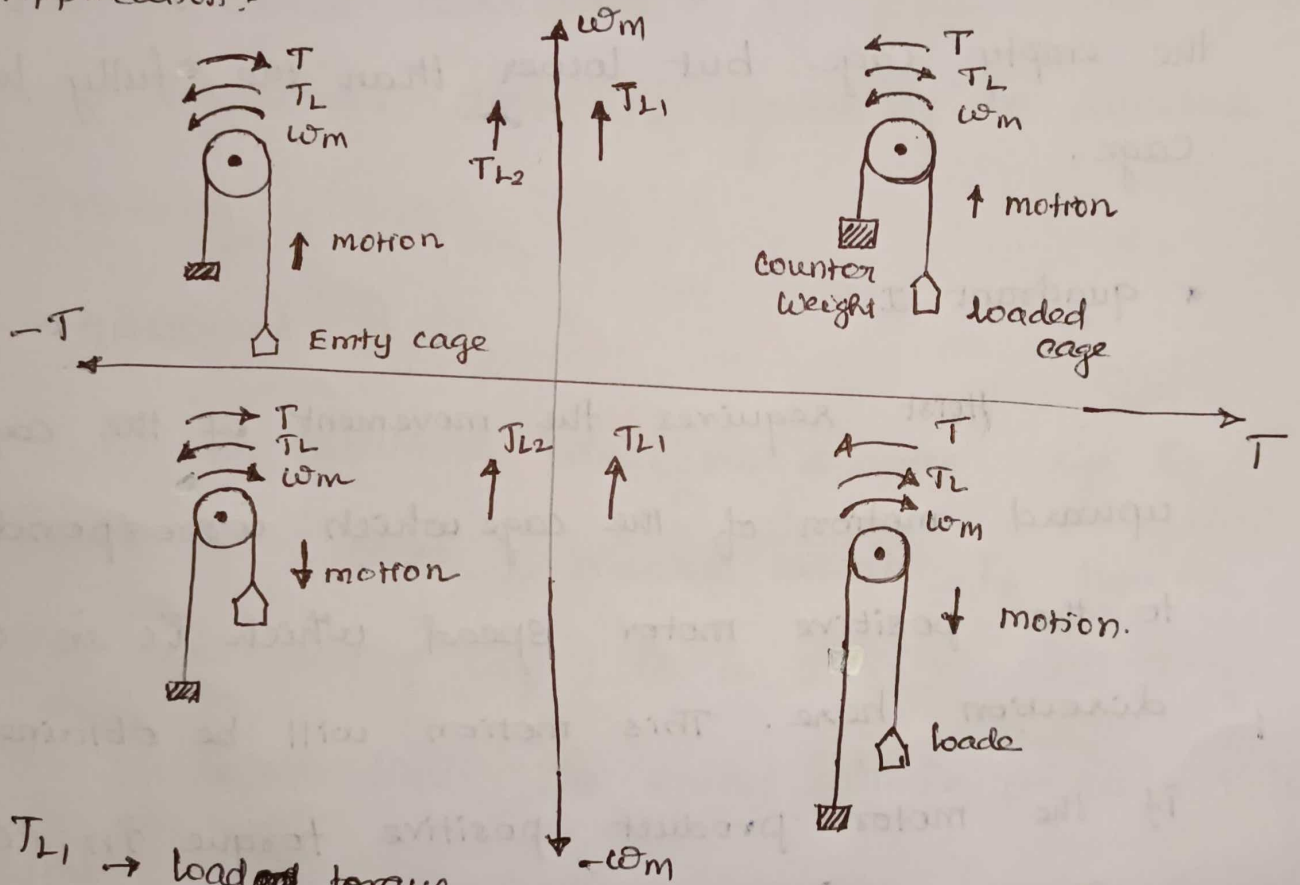
In braking, it works as a generator converting mechanical energy to electrical energy, and thus oppose the motion.



In quadrant I, developed power is positive. Hence machine works as motor supplying mechanical energy. so called 'forward motoring'.

In quadrant 2, power is negative. Hence machine works under braking opposing the motion. so called as 'forward braking'.

Application:-



T_{L1} → loaded torque with loaded cage

T_{L2} → load torque with empty cage.

Direction of motor and load torques, and direction of speed are marked by arrows.

A hoist consists of rope wound on a drum coupled to the motor shaft. One end rope is tied to a cage which is used to transport man or material from one level to another level. Other end of the rope has a counter weight. Weight of the counter weight is chosen to be higher than the weight of the empty cage but lower than the fully loaded cage.

* quadrant I

Hoist requires the movement of the cage upward motion of the cage which corresponds to the positive motor speed which is in CCW direction here. This motion will be obtained if the motor produces positive torque in CCW direction equal to the magnitude of the load torque T_L , since developed power is positive, this is forward motoring operation.

* quadrant - II

The operation is obtained when a loaded

when a loaded cage is lowered. Since the weight of the loaded cage is higher than that of a counter weight. It is able to come down due to gravity itself. In order to limit the speed of a cage within safe value, motor must produce a positive torque (T) equal to T_{L2} in anticlockwise direction. As power and speed are negative, drive is operating in reverse braking.

* quadrant II :-

It is obtained when an empty cage is moved up, since a counter weight is heavier than an empty cage, it is able to pull it up. In order to limit the speed within a safe value motor must produce a braking torque equal to T_{L2} in clockwise direction. Since speed is positive and developed power is negative, it is forward braking operation.

* quadrant III :-

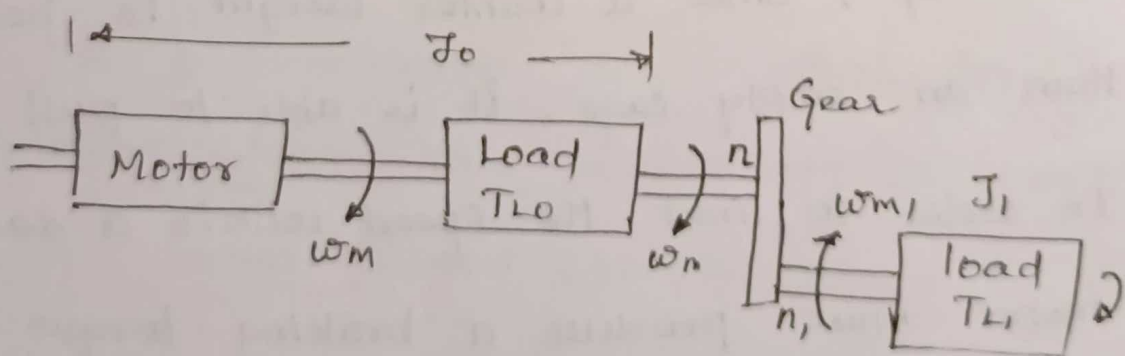
It is obtained, when an empty cage is

lowered. Since an empty cage has a lesser weight than counter weight, the motor should produce a torque in clockwise direction. Since speed is negative and developed power is positive and this is reverse motoring operation.

Equivalent values of drive parameters:

The different parts of a load may be coupled through different mechanisms. These mechanisms are gears, V-belt and crankshaft. The above parts may have different speeds and different types of motions.

Loads with rotational motion:



It consists of motor, two loads and gear. Hence the motor drives two loads. One load directly connected to the shaft and other one through a gear with n and n_1 teeth.

Gear tooth ratio

$$a_1 = \frac{n}{n_1} = \frac{\omega_{m_1}}{\omega_m}$$

losses are neglected in the transmission. Then the kinetic energy due to equivalent inertia is must equal to the kinetic energy of various moving parts.

$$\frac{1}{2} J \omega_m^2 = \frac{1}{2} J_0 \omega_m^2 + \frac{1}{2} J_1 \omega_{m_1}^2$$

$$J \omega_m^2 = J_0 \omega_m^2 + J_1 \omega_{m_1}^2$$

$\frac{1}{\omega_m^2}$, then

$$J = J_0 + J_1 \frac{\omega_{m_1}^2}{\omega_m^2}$$

We know that $a_1 = \frac{\omega_{m_1}}{\omega_m}$, then

$$J = J_0 + a_1^2 J_1$$

power at the motor = $T_L \omega_m$

$T_L \rightarrow$ total equivalent torque referred to motor shaft

power at load $L_0 = T_{L_0} \omega_m$

power at load $L_1 = \frac{T_{L_1} \omega_{m_1}}{\eta_1}$

$\eta_1 \rightarrow$ transmission efficiency of the gear

power at the loads are equal, then

$$T_L \omega_m = T_{L0} \omega_m + \frac{T_{L1} \omega_{m1}}{\eta_1}$$

$\div \omega_m$, we get .

$$T_L = T_{L0} + \frac{T_{L1}}{\eta_1} \times \frac{\omega_{m1}}{\omega_m}$$

$$\therefore \boxed{T_L = T_{L0} + \frac{T_{L1}}{\eta_1} \times a_1}$$

Finally ,

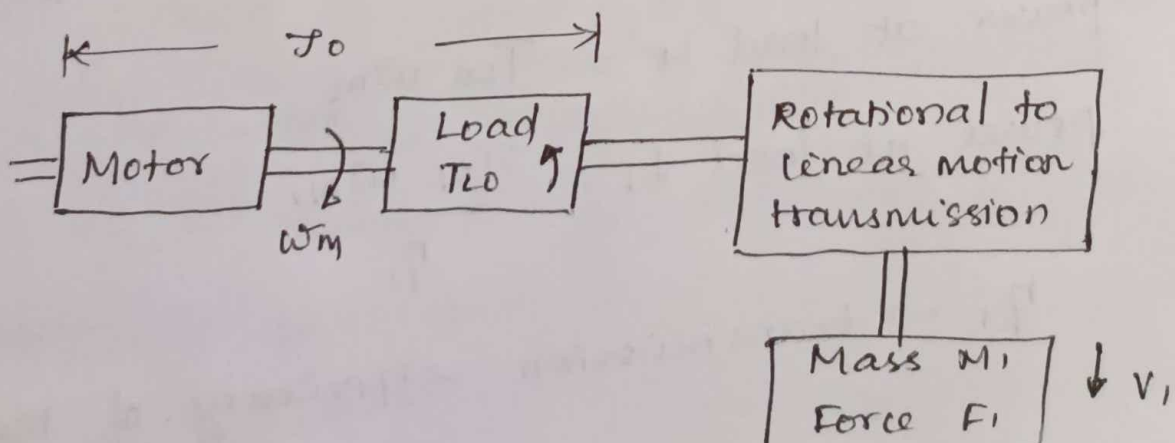
$$J = J_0 + a_1^2 J_1 + a_2^2 J_2 \dots + a_n^2 J_n$$

and

$$T_L = T_{L0} + \frac{a_1 T_{L1}}{\eta_1} + \frac{a_2 T_{L2}}{\eta_2} \dots + \frac{a_n T_{Ln}}{\eta_n}$$

load with Translational Motion:-

consider a motor driving two loads. one is coupled directly to its shaft and other through a transmission system converting rotational motion to linear motion.



The losses are neglected in the transmission system. The kinetic energy is equal to kinetic energy of various moving parts.

$$\frac{1}{2} J \omega_m^2 = \frac{1}{2} J_0 \omega_m^2 + \frac{1}{2} M_1 v_1^2$$

$$J \omega_m^2 = J_0 \omega_m^2 + M_1 v_1^2$$

$$\frac{\circ}{\circ} \omega_m^2, \quad \boxed{J = J_0 + \frac{M_1 v_1^2}{\omega_m^2}}$$

power at the motor = $T_L \omega_m$

power at the load $T_L \omega_m = T_{L0} \omega_m$.

power at the transmission = $\frac{F_1 v_1}{\eta_1}$

Hence the motor and load should be same.

$$T_L \omega_m = T_{L0} \omega_m + \frac{F_1 v_1}{\eta_1}$$

$$\frac{\circ}{\circ} \omega_m, \quad \boxed{T_L = T_{L0} + \frac{F_1 v_1}{\eta_1 \omega_m}}$$

Finally,

$$J = J_0 + M_1 \left(\frac{v_1}{\omega_m} \right)^2 + M_2 \left(\frac{v_2}{\omega_m} \right)^2 + \dots + M_n \left(\frac{v_n}{\omega_m} \right)^2$$

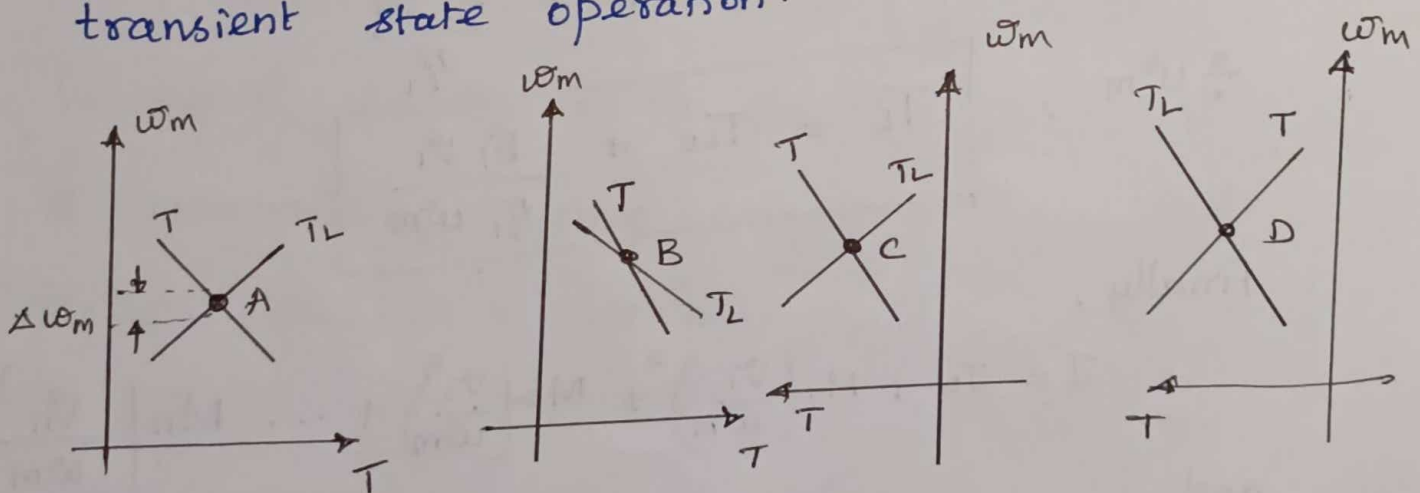
and

$$T_L = T_{L0} + \frac{F_1}{\eta_1} \left(\frac{v_1}{\omega_m} \right) + \frac{F_2}{\eta_2} \left(\frac{v_2}{\omega_m} \right) + \dots + \frac{F_n}{\eta_n} \left(\frac{v_n}{\omega_m} \right)$$

Steady state stability :-

Equilibrium speed of motor-load system can be obtained when motor torque equals the load torque. Electric drive system will operate in steady state at this speed, provided it is the speed of stable state equilibrium.

In most of the electrical drives, the electrical time constant of the motor is negligible compared to the mechanical time constant. During transient condition, electrical motor can be assumed to be in electrical equilibrium implying that steady state speed torque curve also applicable to transient state operation.



Consider, the steady state stability of equilibrium point A. The equilibrium point will be termed as stable state when the operation

will be restored to it after a small departure from it due to disturbance in the motor or load.

Due to disturbance a reduction of $\Delta\omega_m$ in speed. At new speed, electrical motor torque is greater than the load torque, consequently, motor will accelerate and operation will be restored to point A.

Similarly an increase of $\Delta\omega_m$ in speed caused by a disturbance will make load torque greater than the motor torque, resulting into deceleration and restoration of operation to point A. Hence the electric drive is steady state stable at point 'A'.

Equilibrium point B, C, D is obtained when the same motor drives another load. Here the equilibrium point move away when changes in speed. Thus the point B, C and D are an unstable point of equilibrium.

When at equilibrium point following condition is satisfied.

$$\frac{dT_L}{d\omega_m} > \frac{dT}{d\omega_m}$$

Let small disturbance in speed $\Delta\omega_m$ results in perturbations in T and T_L respectively, then

$$(T + \Delta T) = (T_L + \Delta T_L) + J \frac{d(\omega_m + \Delta\omega_m)}{dt}$$

$$T + \Delta T = T_L + \Delta T_L + J \frac{d\omega_m}{dt} + J \frac{d\Delta\omega_m}{dt} \quad \text{--- ①}$$

and we know that

$$T = T_L + J \frac{d\omega_m}{dt} \quad \text{--- ②}$$

subtract eqn ① & ②, we get

$$J \frac{d\Delta\omega_m}{dt} = \Delta T - \Delta T_L \quad \text{--- ③}$$

For very low perturbations, the speed torque curves of the motor and load system can be assumed to be straight line.

$$\text{Thus, } \Delta T = \left(\frac{dT}{d\omega_m} \right) \Delta\omega_m \quad \text{--- ④}$$

$$\Delta T_L = \left(\frac{dT_L}{d\omega_m} \right) \Delta\omega_m. \quad \text{--- ⑤}$$

$\frac{dT}{d\omega_m}$ & $\frac{dT_L}{d\omega_m}$ are slope of steady state speed torque curve.

eqn ④ & ⑤ in eqn ③.

$$J \frac{d\Delta\omega_m}{dt} = \left(\frac{dT}{d\omega_m} \right) \Delta\omega_m - \left(\frac{dT_L}{d\omega_m} \right) \Delta\omega_m.$$

$$J \frac{d\Delta\omega_m}{dt} + \left[\frac{dT_L}{d\omega_m} - \frac{dT}{d\omega_m} \right] \Delta\omega_m = 0$$

It is first order differential equation. If initial deviation in speed at $t=0$ be $(\Delta\omega_m)_0$ then the solution of differential equation.

$$\Delta\omega_t = (\Delta\omega_m)_0 \exp \left[-\frac{1}{J} \left\{ \frac{dT_L}{d\omega_m} - \frac{dT}{d\omega_m} \right\} t \right]$$

The system operating point will be stable when $\Delta\omega_m$ approaches to zero as t approaches infinity.

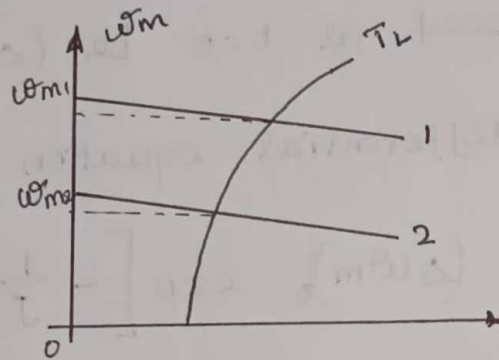
Modes of operation of Electric Drives:-

- a) steady state
- b) Acceleration including starting
- c) Acceleration including stopping.

Steady state operation:-

Steady state operation is achieved when motor torque equals to load torque. change in motor speed is achieved by varying the steady state motor speed torques are equal at this speed. then motor torque equals to load torque at the new desired speed.

When the electric motor parameters are adjusted to provide speed torque curve 1 drives runs at the desired speed ω_{m1} .



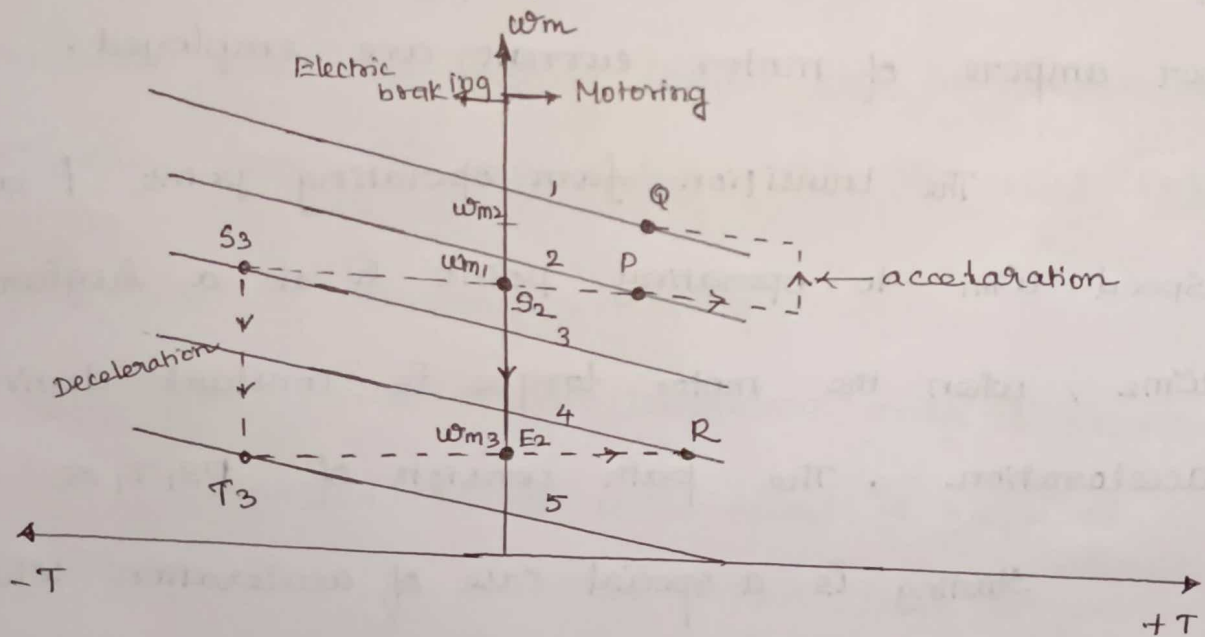
Now motor speed is changed to ω_{m2} when the motor parameters are adjusted to provide speed torque curve 2.

When the load torque opposes the motion, the machine works as a motor operating in quadrant I or III depending on the direction of rotation.

Steady state operation for a case can be obtained by adding mechanical brake which will produce a torque in a direction to oppose the motion. The steady state stable operation is obtained at the speed for which braking torque is equal to load torque. Now drive operates in quadrant II or IV depending on the direction of rotation.

2) Acceleration Including Starting:-

Acceleration and deceleration modes are transient operations of the motor. Electric drives operate in acceleration mode whenever an increase in its speed is required. For this electric motor speed torque curve must be changed so that motor torque exceeds the load torque. Time taken for a given change in motor speed depends on inertia of motor-load system and the amount by which motor torque exceeds the load torque.



Increase in electric motor torque is accompanied by an increase in motor current. Care must be taken to restrict the motor current with in a value which is safe for both motor and power converter. In applications involving acceleration periods of long duration, current

must not be allowed to exceed the rated value of the drive motor.

When acceleration conditions are short duration a current higher than the rated value is allowed during acceleration. In closed loop electric drives requiring fast response motor current may be intentionally forced to the maximum value in order to achieve high acceleration. Torque produced by AC motor for a given current is usually a function of motor control method employed. In high performance of electric drives, methods which produce high torque per ampere of motor current are employed.

The transition from operating point P at speed ω_{m1} to operating point Q at a higher speed ω_{m2} , when the motor torque is constant during acceleration. The path consists of P, T, Q

Starting is a special case of acceleration where a motor speed change from 0 to a desired speed takes place. All points mentioned in relation to acceleration are applicable to starting of the motor. Maximum current allowed should not only be safe for motor and power converter but the drop in source voltage caused

due to it should also be in acceptable limits.

In ac motor starting torque per ampere has different values of various starting methods.

When starting takes place at no load or light load condition, the method with low starting torque can be employed. When the electric motor must start with substantial load torque or when fast is required, methods with high starting torque must be used.

In some application, the electric motor should accelerate smoothly, without any jerk. This can be obtained when the starting torque can be increased steplessly from its zero value. This is called soft start.

Deceleration Including Stopping:-

Motor operation in deceleration mode is required when a decrease in its motor speed is required.

Deceleration can be obtained when load torque exceeds the motor torque.

When load torque is always present with substantial magnitude, enough deceleration can be achieved by simply reducing the motor torque value.

when load torque may not always have substantial amount or where simply reducing the motor torque to zero does not provide enough deceleration, mechanical brakes can be used to produce the required magnitude of deceleration or electric braking can be employed. Now both motor and load torque oppose the motion, thus producing larger deceleration. During electric braking condition, motor current tends to exceed the safe limit. Some control are made to ensure that the current is restricted within safe limit.

During transition from point 'P' at speed ω_m to a point 'R' at a lower speed ω_m . when deceleration mode is carried out using electric braking at a constant braking torque, the operating points moves along the path $P S_3 T_3 R$

when sufficient load torque is present or when mechanical braking is used the operation takes place along the path $P S_2 T_2 R$. stopping is a special case of deceleration mode where the speed of the running motor is changed to zero. In most of the applications requiring frequent, quick, accurate or rapid emergency stops, the electric braking is mainly used.

A motor drives two loads, one has rotational motion. It is coupled to the motor through a reduction gear with $a = 0.1$ and efficiency of 90%. The load has a moment of inertia of 10 kg-m^2 and the torque of 10 N-m . Other load has translational motions and consists of 1000 kg weight to be lifted up at a uniform speed of 1.5 m/s . Coupling between this load and the motor has an efficiency of 85%. Motor has an inertia of 0.2 kg-m^2 and runs at a constant speed of 1420 rpm . Determine equivalent inertia referred to the motor shaft and power developed by the motor.

Solution:.

The total moment of inertia referred to the motor shaft

$$J = J_0 + a_1^2 J_1 + M_1 \left(\frac{V_1}{\omega} \right)^2$$

$$J = 0.2 \text{ kg-m}^2, \quad a_1 = 0.1, \quad J_1 = 10 \text{ kg-m}^2$$

$$V = 1.5 \text{ m/s} \quad \omega_m = \frac{1420 \times \pi}{30} = 148.7 \text{ rad/sec.}$$

$$J = 0.2 + (0.1)^2 \times 10 + 1000 \left(\frac{1.5}{148.7} \right)^2$$

$$\boxed{J = 0.4 \text{ kg-m}^2}$$

$$T_L = \frac{a T_{L1}}{n_1} + \frac{F_1}{n_1'} \left(\frac{V_1}{\omega_m} \right)$$

$$n_1 = 0.9, \quad a_1 = 0.1, \quad T_{L1} = 10 \text{ N-m} \quad n_1' = 0.85$$

$$F_1 = 1000 \times 9.81$$

$$v_1 = 1.5 \text{ m/s}$$

$$\omega_m = 148.7 \text{ rad/sec}$$

$$T_L = \frac{0.1 \times 10}{0.9} \times \frac{1000 \times 9.81}{0.85} \left(\frac{1.5}{148.7} \right)$$

$$T_L = 117.53 \text{ N-m}$$

An electric drives has the following parameters,
 $J = 10 \text{ kg-m}^2$, $T = 100$ to 0.1 N , Passive load torque
 $T_L = 0.05 \text{ N}$, Initially the driving is operating in
Steady state. Now it is to be reversed. For this
motor characteristics is changed to $T = -100 - 0.1 \frac{N}{N}$
calculate the time reversal.

For steady state

$$T = T_L$$

$$T - T_L = 0$$

$$100 - 0.1N - 0.05N = 0$$

$$0.15N = 100$$

$$N = \frac{100}{0.15} = 666.7 \text{ rpm}$$

After reversal

$$-100 - 0.1N - 0.05N = 0$$

$$N = -666.7 \text{ rpm}$$

When reversing, $J \frac{d\omega_m}{dt} = -100 - 0.1N - 0.05N$

$$\frac{dN}{dt} = \frac{30}{J\pi} (-100 - 0.15N)$$

$$\frac{dN}{dt} = -95.49 - 0.143N$$

$$t = \int dt = \int_{N_1}^{N_2} \frac{dN}{-95.49 - 0.143N}$$

where $N_1 = 666.7 \text{ rpm}$

$$N_2 = 0.95 \times 666.7$$

$$= 633.4 \text{ rpm}$$

After integration

$$t = 25.58 \text{ sec}$$

Rectifier control of DC Drives

Single Phase Fully controlled ~~bridge~~ rectifier fed separately excited

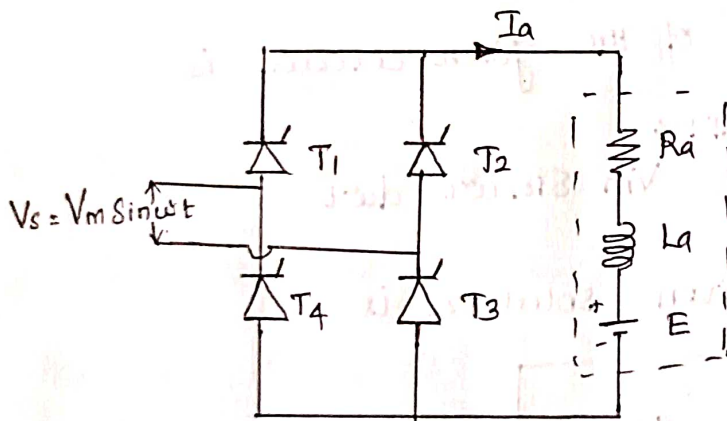
DC motor:-

The input ac voltage of the rectifier circuit is given by

$$V_s = V_m \sin \omega t$$

The armature circuit of given DC motor is consists of armature resistance R_a and inductance L_a respectively and 'E' is the back emf of the motor.

Rectifier circuit Diagram:-



The operation of the rectifier circuit based on two modes of operations.

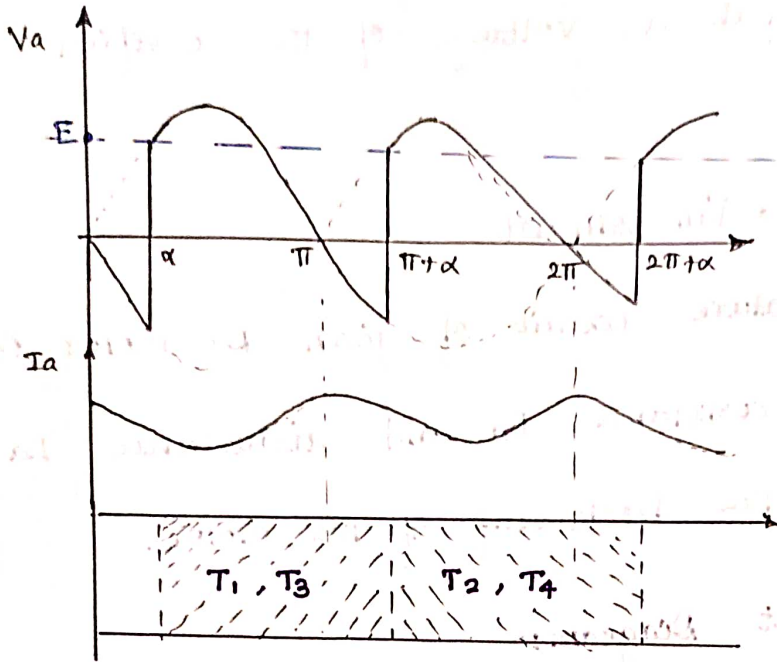
- (i) continuous conduction mode
- (ii) Discontinuous conduction mode.

(i) Continuous conduction Mode:-

At α , T_1 and T_3 are triggered and continuously conduct upto $\pi + \alpha$.

At $\pi + \alpha$, T_2 and T_4 are triggered and continuously conduct upto $2\pi + \alpha$. when the armature current flows continuously, the conduction is said to be continuous.

output waveform:



The armature voltage of the given circuit is

$$V_a = \frac{1}{\pi} \int_{\alpha}^{\pi + \alpha} V_m \sin \omega t \, d\omega t$$

integrated, the given solution is

$$V_a = \frac{2V_m}{\pi} \cos \alpha$$

where $V_a \rightarrow$ armature terminal voltage

In separately excited motor,

$$V_a = E + I_a R_a$$

and also $T \propto I_a$

$$T = K I_a$$

Similarly Back emf, $E = \frac{\Phi_z N}{60} \left(\frac{P}{A} \right)$

Then $E \propto \omega_m$

$$E = k \omega_m$$

$$\therefore V = k \omega_m + \frac{I}{k} R_a$$

$$V - \frac{I}{k} R_a = k \omega_m$$

$$\omega_m = \frac{V}{k} - \frac{I}{k^2} R_a$$

For continuous conduction,

$\rightarrow \alpha < \pi/2$, E should be greater than or equal to V_m

$\rightarrow \alpha > \pi/2$, E should be greater than or equal to $V_m \sin \alpha$

\therefore The ideal no load speed is given by

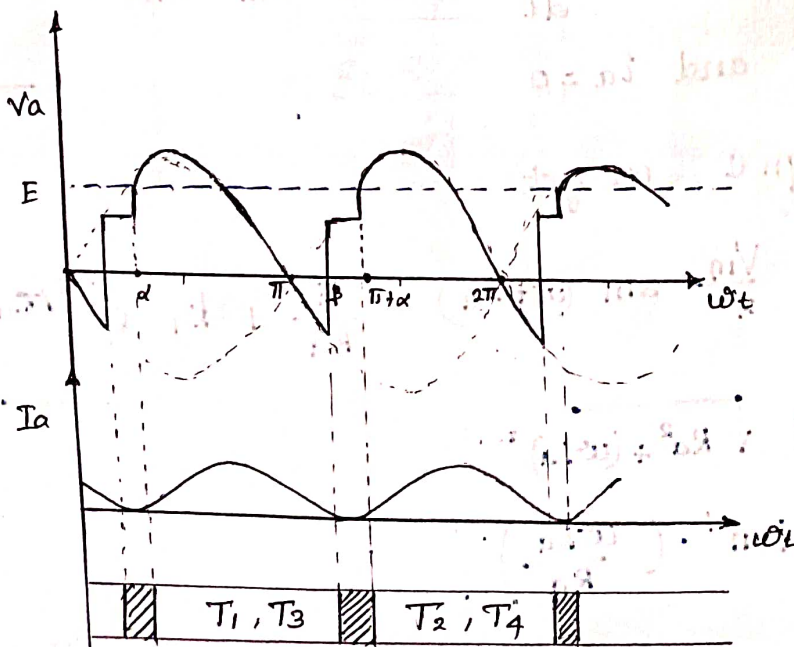
$$\omega_{m0} = \frac{V_m}{k}, \text{ for } 0 \leq \alpha \leq \pi/2$$

$$\omega_m = \frac{V_m \sin \alpha}{k}, \text{ for } \pi/2 \leq \alpha \leq \pi$$

(ii) Discontinuous conduction mode:

when the armature current does not flow continuously,

the motor is said to be discontinuous conduction.



In discontinuous conduction mode, current starts flow up T_1 and T_3 at $\omega t = \alpha$, motor gets connected to the source and its terminal voltage is V_s .

When $\omega t = \pi$, the current falls down to zero at $\omega t = \beta$ because the absence of current in T_1 and T_3 at turned off conditions.

At $\omega t = \pi + \alpha$, T_2 and T_4 are fired and the next cycle of the motor terminal voltage V_s starts.

In the drive operation in two intervals.

(i) Duty interval ($\alpha \leq \omega t \leq \beta$)

The motor is connected to the source and $V_a = V_s$

(ii) Zero current interval ($\beta \leq \omega t \leq \pi + \alpha$)

Now $V_a = E$ and $i_a = 0$ for this interval.

The armature voltage,

$$V_a = R_a i_a + L_a \frac{di_a}{dt} + E = V_m \sin \omega t \quad \text{--- (1)}$$

$$V_a = E \quad \text{and} \quad i_a = 0 \quad \text{--- (2)}$$

Solving eqn (1), we get

$$i_a(\omega t) = \frac{V_m}{z} \sin(\omega t - \phi) - \frac{E}{R_a} + k_1 e^{-t/\tau_a}$$

$$\text{and } z = \sqrt{R_a^2 + (\omega L_a)^2} \quad \text{--- (3)}$$

$$\phi = \tan^{-1} \left(\frac{\omega L_a}{R_a} \right)$$

$$V_a = i_a R_a + L_a \frac{di_a}{dt} + E_b = V_m \sin \omega t \quad \text{--- ①}$$

for $\alpha \leq \omega t \leq \beta$

$$V_a = E_b \text{ and } i_a = 0 \text{ for } \beta \leq \omega t \leq \pi + \alpha \quad \text{--- ②}$$

From eqn ①,

$$R_a i_a + L_a \frac{di_a}{dt} = V_m \sin \omega t - E_b$$

$$\frac{\partial}{\partial L_a} \Rightarrow \frac{R_a}{L_a} i_a + \frac{di_a}{dt} = \frac{V_m \sin \omega t - E_b}{L_a} \quad \text{--- ③}$$

Complex Function (CF)

$$\frac{di_a}{dt} + \frac{R_a}{L_a} i_a = 0$$

$$D i_a + \frac{R_a}{L_a} i_a = \frac{V_m \sin \omega t - E_b}{L_a}$$

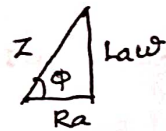
$$\left[D + \frac{R_a}{L_a} \right] i_a = \frac{V_m \sin \omega t - E_b}{L_a} \quad \because D = \frac{d}{dt}$$

Using complex function and particular integral (PI) in the above equation, we get

$$\left[D + \frac{R_a}{L_a} \right] i_a = 0$$

$$\begin{aligned} \text{C.F.} &= C_1 e^{-\frac{R_a}{L_a} t} \\ &= C_1 e^{-\frac{R_a \cdot \omega}{L_a \cdot \omega} t} \end{aligned}$$

$$\boxed{\text{C.F.} = C_1 e^{-\omega t \cot \phi}} \quad \text{④}$$



$$Z^2 = R_a^2 + L_a \omega^2$$

$$\begin{aligned} \tan \phi &= \frac{X_L}{R_a} \\ &= \frac{L_a \omega}{R_a} \end{aligned}$$

$$\cot \phi = \frac{R_a}{L_a \omega}$$

$$\text{PI:} \quad \left[D + \frac{R_a}{L_a} \right] i_a = \frac{V_m \sin \omega t}{L_a}$$

$$\text{PI}_1 = \frac{V_m \sin \omega t}{L_a \left[D + \frac{R_a}{L_a} \right]} = \frac{V_m \sin \omega t}{L_a \left[D^2 - \frac{R_a^2}{L_a^2} \right]} \times \left[D - \frac{R_a}{L_a} \right]$$

$$\begin{aligned}
 &= \frac{V_m \left[\omega \cos \omega t - \frac{R_a}{L_a} \sin \omega t \right]}{L_a \left[-\omega^2 - \frac{R_a^2}{L_a^2} \right]} \\
 &= \frac{V_m \left[\omega \cos \omega t - \frac{R_a}{L_a} \sin \omega t \right]}{-L_a \left[\frac{R_a^2 + (\omega L_a)^2}{L_a^2} \right]} \\
 &= \frac{V_m L_a \left[\frac{R_a}{L_a} \sin \omega t - \omega \cos \omega t \right]}{R_a^2 + (\omega L_a)^2} \\
 &= \frac{V_m L_a \left[\frac{R_a}{L_a} \sin \omega t - \omega \cos \omega t \right]}{z^2}
 \end{aligned}$$

where $z^2 = R_a^2 + (\omega L_a)^2$

$$\begin{aligned}
 &= \frac{V_m}{z} \left[\frac{R_a}{z} \sin \omega t - \frac{\omega L_a \cos \omega t}{z} \right] \\
 &= \frac{V_m}{z} \left[\sin \omega t \cos \phi - \cos \omega t \sin \phi \right]
 \end{aligned}$$

where $\cos \phi = \frac{R_a}{z}$, $\sin \phi = \frac{\omega L_a}{z}$

$$\therefore \boxed{P I_1 = \frac{V_m}{z} \left[\sin(\omega t - \phi) \right]} \quad \text{--- (5)}$$

$P I_2$:

$$\left[D + \frac{R_a}{L_a} \right] i_a = -\frac{E_b}{L_a}$$

$$\begin{aligned}
 P I_2 &= \frac{-E_b}{L_a \left[D + \frac{R_a}{L_a} \right]} = \frac{-E_b}{L_a \times \frac{R_a}{L_a} \left[1 + \frac{D L_a}{R_a} \right]} \\
 &= \frac{-E_b}{R_a} \left[1 + \frac{D L_a}{R_a} \right] \\
 &= \frac{-E_b}{R_a} + 0
 \end{aligned}$$

$$\boxed{PI_2 = -\frac{E_b}{R_a}} \quad \text{--- (6)}$$

from equation (4) to (6)

$$i_a(\omega t) = CF + PI_1 + PI_2$$

$$i_a(\omega t) = C_1 e^{-\omega t \cot \varphi} + \frac{V_m}{\alpha} \sin(\omega t - \varphi) - \frac{E_b}{R_a} \quad \text{--- (7)}$$

$$\text{where } \alpha = \sqrt{R_a^2 + (\omega L_a)^2}$$

$$\varphi = \tan^{-1} \left(\frac{\omega L_a}{R_a} \right)$$

C_1 evaluated by using initial value condition $i_a(\alpha) = 0$ in eqn (7),

$$0 = C_1 e^{-\alpha \cot \varphi} + \frac{V_m}{\alpha} \sin(\alpha - \varphi) - \frac{E_b}{R_a}$$

$$C_1 e^{-\alpha \cot \varphi} = - \left[\frac{V_m}{\alpha} \sin(\alpha - \varphi) - \frac{E_b}{R_a} \right] \quad \text{--- (8)}$$

sub in eqn (7),

$$i_a(\omega t) = \left[\frac{V_m}{\alpha} \sin(\omega t - \varphi) - \frac{E_b}{R_a} \right] - \left[\frac{V_m}{\alpha} \sin(\alpha - \varphi) - \frac{E_b}{R_a} \right] e^{-\omega t \cot \varphi}$$

$$i_a(\omega t) = \left[\frac{V_m}{\alpha} \sin(\omega t - \varphi) - \frac{E_b}{R_a} \right] - \left[\frac{V_m}{\alpha} \sin(\alpha - \varphi) - \frac{E_b}{R_a} \right] \cdot e^{-(\omega t - \alpha) \cot \varphi}$$

since $i_a(\beta) = 0$,

$$0 = \left[\frac{V_m}{\alpha} \sin(\beta - \varphi) - \frac{E_b}{R_a} \right] - \left[\frac{V_m}{\alpha} \sin(\alpha - \varphi) - \frac{E_b}{R_a} \right] e^{-(\beta - \alpha) \cot \varphi} \quad \text{--- (9)}$$

$\beta \rightarrow$ can be evaluated by iterative solution of equation (10)

$$V_a = E + I_a R_a$$

Armature voltage

$$V_a = \frac{1}{\pi} \left[\int_{\alpha}^{\beta} V_m \sin \omega t \, d\omega t + \int_{\beta}^{\pi+\alpha} E \, d\omega t \right]$$

$$V_a = \frac{V_m (\cos \alpha - \cos \beta) + (\pi + \alpha - \beta) E}{\pi}$$

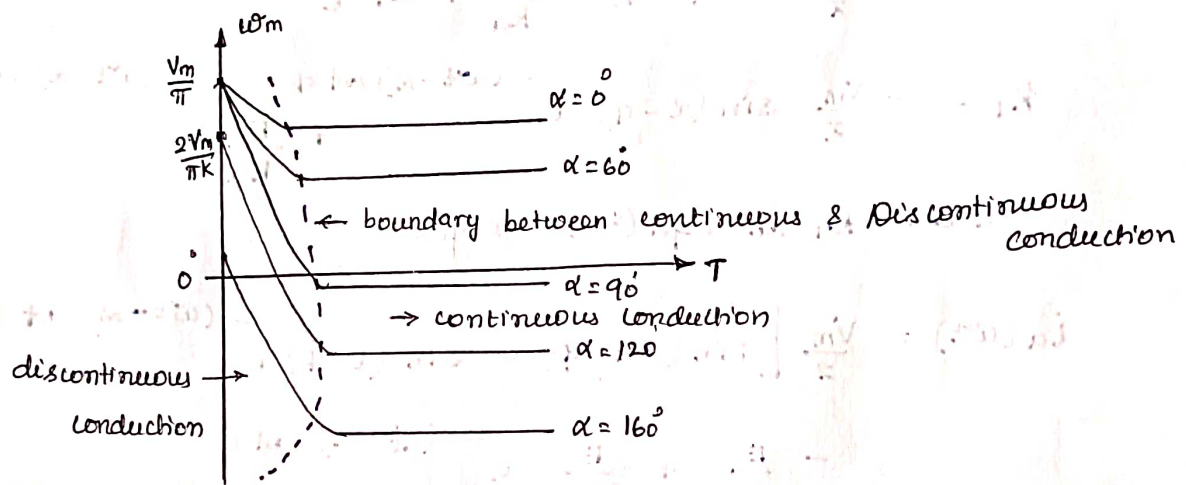
$$E = k \omega_m, \quad T = k I_a$$

$$\omega_m = \frac{V_m [\cos \alpha - \cos \beta]}{k [\beta - \alpha]} - \frac{\pi R_a}{k^2 (\beta - \alpha)} T$$

where $\beta = \pi + \alpha$ substitute in eqn (5) and find critical value of speed ω_{mc} which separates from continuous conduction to discontinuous conduction for given α .

$$\omega_{mc} = \frac{R_a V_m}{\alpha k} \sin(\alpha - \phi) \left[\frac{1 + e^{-\pi \cot \phi}}{e^{-\pi \cot \phi} - 1} \right]$$

Boundary condition between continuous conduction & discontinuous conduction:

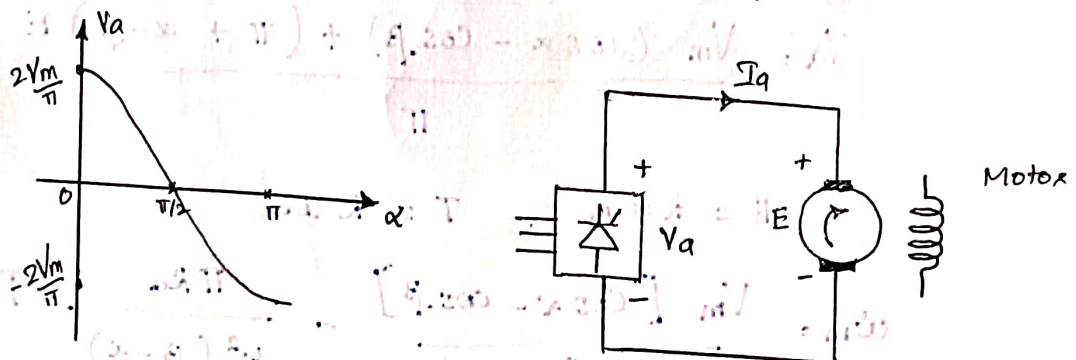


In discontinuous conduction, the speed regulation is poor.

In continuous conduction, the speed regulation is good.

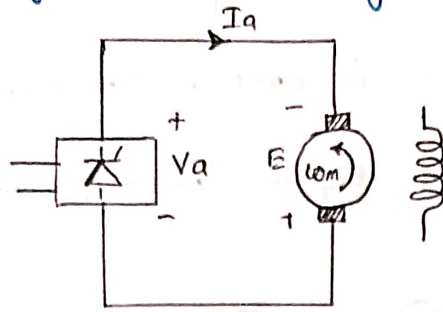
In continuous conduction, for a given α , any increase in torque causes ω_m and E to drop. So I_a & T can increase

In discontinuous conduction, any increase in torque I_a causes β to increase and V_a to drop.



$\alpha \geq 90^\circ$, $\omega_m > 0$, motoring.

The drive operates in quadrant I [forward motoring] & IV
[reverse generative braking]

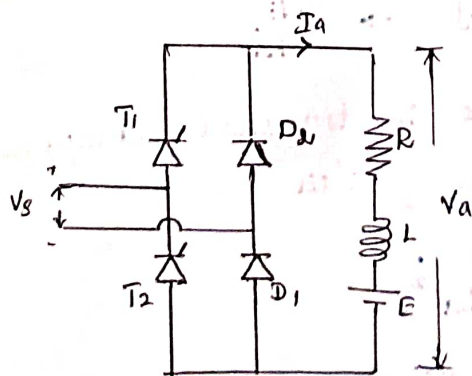


$$\alpha > 90, \omega_m < 0$$

Under continuous conduction, in first quadrant ω_m is positive, $\alpha \leq 90^\circ$, V_a & E is positive. For positive I_a , rectifier to deliver power and the motor to continue it, thus giving forward motoring.

In quadrant IV, E has reversed due to reversal of ω_m . But I_a still in same direction, machine works as a generator producing braking torque. So rectifier takes power from dc terminals and transfer it to ac mains.

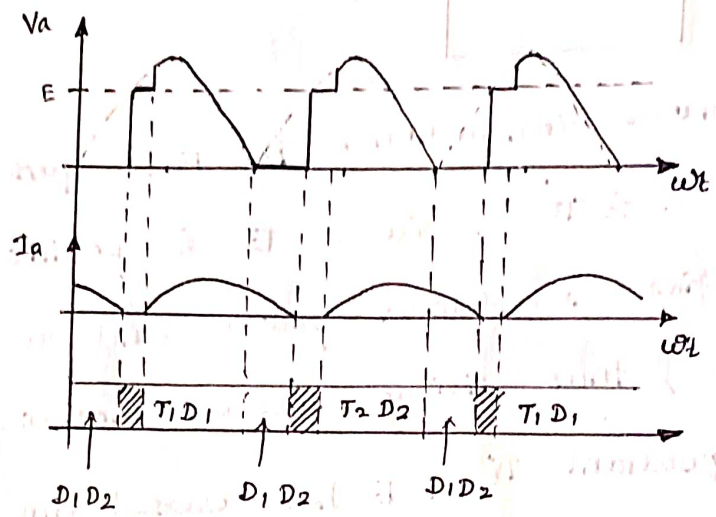
Single Phase Half controlled Rectifier control of dc separately excited Motor.



Discontinuous conduction:-

At α , T_1 is fired and motor gets connected to the source through T_1 and D_1 and $V_a = V_s$

At π , armature flows through D_2 and D_1 get forward biased and freewheels through the path D_1 and D_2 , and the motor terminal voltage is zero. So maintain the constant level of battery before T_2 is fired.



(i) Duty interval ($\alpha \leq \omega t \leq \pi$)

$$I_a R_a + L_a \frac{di_a}{dt} + E = V_s = V_m \sin \omega t$$

$$i_a(\omega t) = \frac{V_m}{Z} \sin(\omega t - \phi) - \frac{E}{R_a} + k_1 e^{-t/\tau_a}$$

The armature current has two components.

a) due to ac source $\rightarrow \frac{V_m}{Z} \sin(\omega t - \phi)$

b) due to back emf $\rightarrow -\frac{E}{R_a}$

Each of the components is transient component & can be represented by $k_1 e^{-t/\tau_a}$

$$Z = \sqrt{R_a^2 + (\omega L_a)^2}$$

$$\phi = \tan^{-1} \left(\frac{\omega L_a}{R_a} \right)$$

$$i_a(\omega t) = \frac{V_m}{Z} \left[\sin(\omega t - \phi) - \sin(\alpha - \phi) e^{-(\omega t - \alpha) \cot \phi} \right] - \frac{E}{R_a} \left[1 - e^{-(\omega t - \alpha) \cot \phi} \right] \quad \text{--- (1)}$$

$\omega t = \pi$, $i_a(\pi)$ is found which is the initial position for the next interval.

(ii) Free wheeling Interval $[\pi \leq \omega t \leq \beta]$

$$i_a R_a + L_a \frac{di_a}{dt} + E = 0$$

The solution of the above equation with initial condition of $i_a(\pi)$, will give

$$i_a(\omega t) = \frac{V_m}{Z} \left[\sin \phi \cdot e^{-(\omega t - \pi) \cot \phi} - \sin(\alpha - \phi) e^{-(\omega t - \alpha) \cot \phi} \right] - \frac{E}{R_a} \left[1 - e^{-(\omega t - \alpha) \cot \phi} \right] \quad \text{--- (2)}$$

for $\pi \leq \omega t \leq \beta$.

(iii) Zero current Interval $(\beta \leq \omega t \leq \pi + \alpha)$

$$V_a = E, \quad i_a = 0$$

Substitute $\omega t = \beta$ and equating $i_a(\beta) = 0$ in eqn (2),

$$e^{\beta \cot \phi} = \frac{R_a V_m}{Z k} \left[\sin \phi e^{\pi \cot \phi} - \sin(\alpha - \phi) e^{\alpha \cot \phi} \right] + e^{\alpha \cot \phi} \quad \text{--- (3)}$$

$V_a \rightarrow$ average voltage across the armature.

$$V_a = \frac{1}{\pi} \left[\int_{\alpha}^{\pi} V_m \sin \omega t d(\omega t) + \int_{\beta}^{\pi + \alpha} E d\omega t \right]$$

$$V_a = \frac{V_m (1 + \cos \alpha) + (\pi + \alpha - \beta) E}{\pi}$$

$$E = k \omega_m, \quad T = k I_a$$

$$V = E + I_a R_a$$

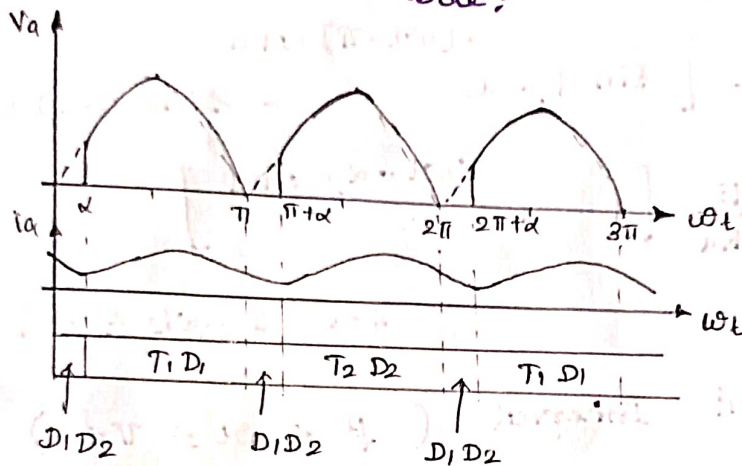
$$\therefore \omega_m = \frac{V}{k} - \frac{R_a}{k^2} T$$

$$\omega_m = \frac{V_m (1 + \cos \alpha)}{k(\beta - \alpha)} - \frac{\pi R_a}{k^2 (\beta - \alpha)} \cdot T$$

$\beta = \pi + \alpha$ sub. in eqn (3), gives the critical speed

$$\omega_{mc} = \frac{R_a V_m}{2k} \left[\frac{\sin \phi \cdot e^{-\alpha \cot \phi} - \sin(\alpha - \phi) e^{-\pi \cot \phi}}{1 - e^{-\pi \cot \phi}} \right]$$

Continuous Conduction Mode:



At α , T_1 is fired, the power gets connected to the source. So current flows through T_1 and D_1 and $v_a = V_s$

At π , D_2 gets forward biased and armature current freewheels through the path D_1, D_2

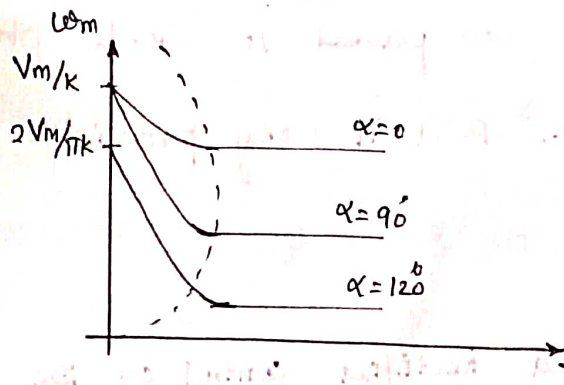
At $\pi + \alpha$, T_2 is fired, the armature current flows through T_2 and D_2 .

$$V_a = \frac{1}{\pi} \int_{\alpha}^{\pi} V_m \sin \omega t \, d(\omega t)$$

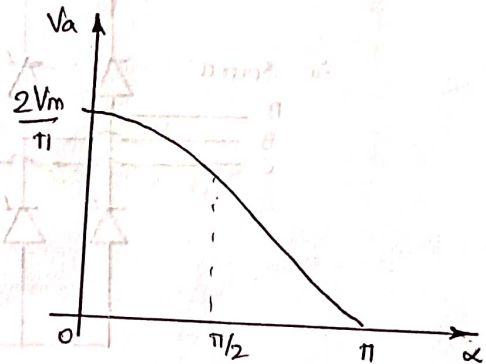
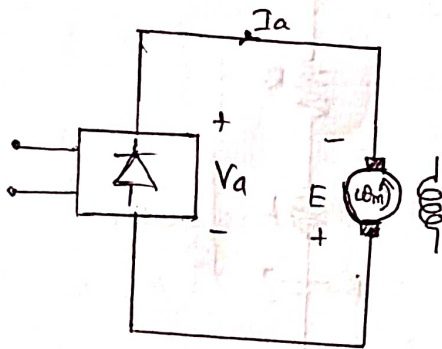
$$V_a = \frac{V_m}{\pi} (1 + \cos \alpha)$$

$$\omega_m = \frac{V_m}{\pi k} (1 + \cos \alpha) - \frac{R_a}{k^2} \cdot T$$

Speed - Torque Curve :



In operates only in 1st quadrant and not operate in quadrant IV, because the output voltage cannot reversed.



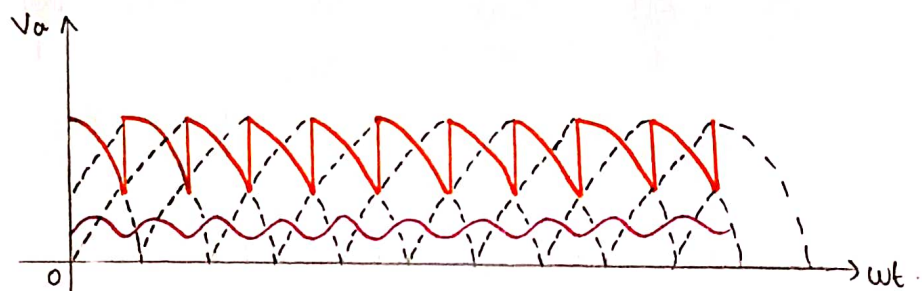
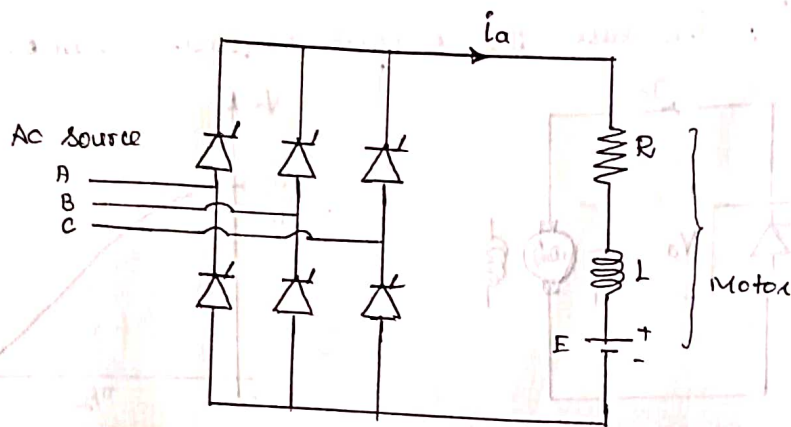
when half controlled rectifier is coupled to an active load, the motor speed can reverse, reverse the E , But current direction doesn't change, machine works as a generator producing braking torque. Since rectifier voltage cannot reverse, generated energy cannot be transferred to ac source, and its observed in the armature circuit.

Such braking operation, produces the large current flow through rectifier and motor. Since it cannot be regulated by adjustment of firing angle, it will damage the rectifier and motor. So take more care to avoid such operation. If such operation cannot be avoided, fully controlled should be used.

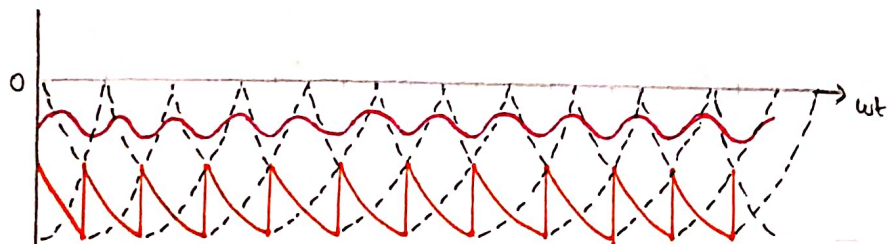
One
Three

A half wave single phase rectifier is cheaper and gives high power factor compared to single phase fully controlled rectifier. But it only provides control in quadrant - I.

Three phase Fully controlled Rectifier control of DC separately excited motor:



T_1	T_3	T_5	T_1	T_3	T_5
T_6	T_2	T_4	T_6	T_2	T_4



T_3	T_5	T_1	T_3	T_5	T_1
T_4	T_6	T_2	T_4	T_6	T_2

Three phase fully controlled rectifier fed into separately excited dc motor. The numbering of SCR's is 1, 3, 5 for the positive group and 4, 6, 2 for the negative group

T_1 connected to phase A can't be fired below an angle of 30° . because its reverse biased by an already conducting SCR. Hence minimum firing angle is $\pi/6$

Positive group of SCR's are fired at an interval is 120° similarly negative group of SCR's are fired at an angle is 120° . But both SCR's are fired at an interval 60° .

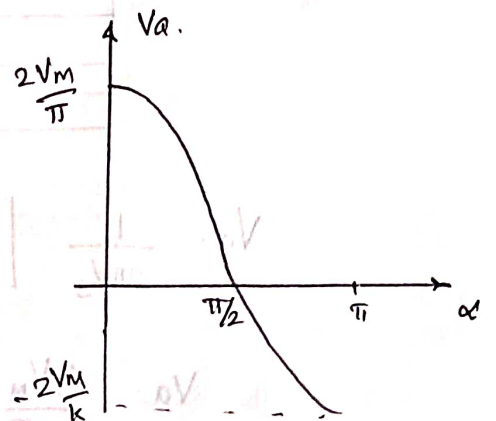
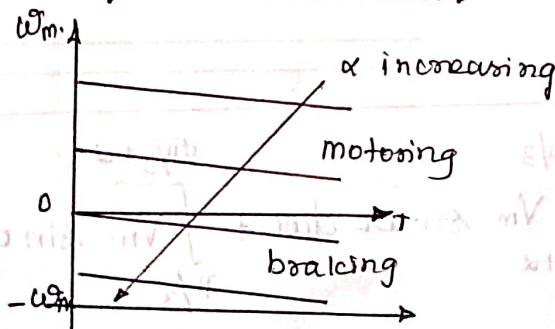
For continuous conduction mode ($\alpha < 60^\circ$) $\alpha + 2\pi/3$

the average output voltage = $\frac{1}{\pi/3} \int_{\alpha + \pi/3} V_m \sin \omega t d(\omega t)$

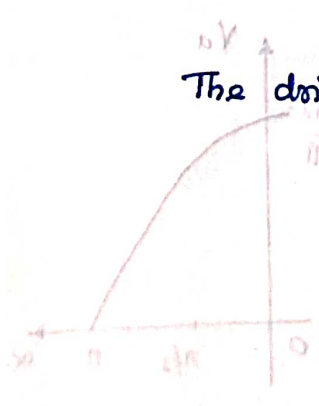
$$V_a = \frac{3}{\pi} V_m \cos \alpha$$

$$\omega_m = \frac{2V_m}{\pi k} \cos \alpha - \frac{R_a}{k^2} T$$

Speed Torque characteristics:-



The drive operation is quadrant I & IV.



CHOPPER CONTROL OF DC DRIVES

Chopper :-

A chopper is a device which is used to convert a fixed DC source voltage to variable DC voltage at the load side. It is inserted inbetween a fixed DC source voltage and the DC armature for its speed control below base speed. The speed of the DC drive can be controlled below the rated speed by supplying variable DC voltages.

Advantages :-

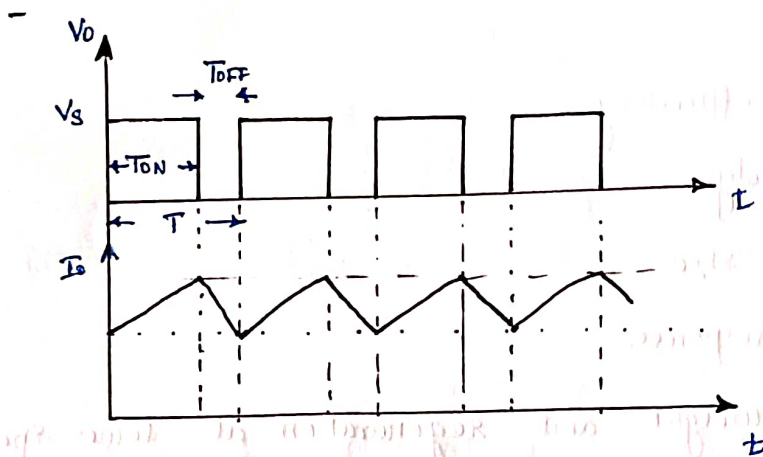
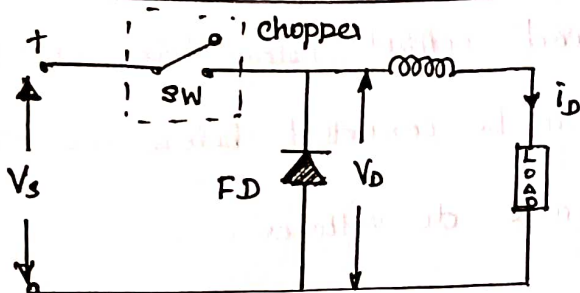
- * High efficiency
- * Flexibility
- * Small size
- * Fast response
- * Less weight and regeneration at lower speed.

Advantages of chopper over DC drive :-

- * Chopper operates at high frequency, so ripple in the armature current is less. So that it reduces the machine losses and also eliminates the discontinuous conduction mode. So improve speed regulation and transient response.

- * The operation of chopper is synchronism with the ac source voltage allows an improvement in the line power factor and a reduction in the armature current ripple.

Introduction to time ratio control and Frequency modulation:



- * During the period T_{on} , chopper is ON and load voltage is equal to source voltage V_s .
- * During the interval T_{off} , chopper is off, load current flows through the freewheeling diode. The load terminals are short circuited by FD and load voltage is zero during T_{off} .

* During T_{ON} , load current rises whereas during T_{OFF} , load current decays,

The average load voltage,

$$V_o = \frac{T_{ON}}{T_{ON} + T_{OFF}} \times V_s$$
$$= \frac{T_{ON}}{T} \times V_s$$

$$\therefore V_o = \alpha V_s$$

where $\alpha \rightarrow$ duty cycle $\therefore \alpha = \frac{T_{ON}}{T}$

$$T = T_{ON} + T_{OFF}$$

$T_{ON} \rightarrow$ on time, $T_{OFF} \rightarrow$ off time.

and also $V_o = f \cdot T_{ON} \times V_s$

$$\therefore f = \frac{1}{T} \rightarrow \text{chopping frequency.}$$

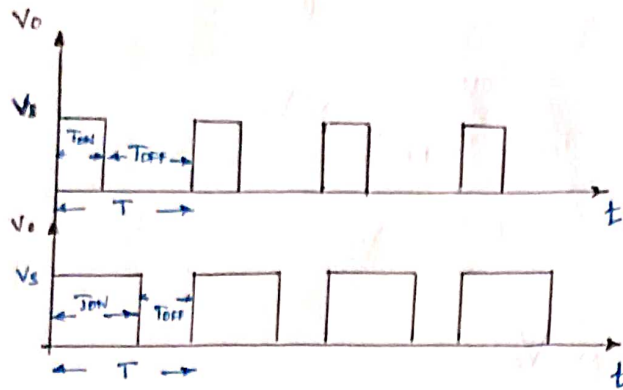
* The switch S can be controlled by varying the duty ratio α and one of the methods is time ratio control. In time ratio control also known as pulse width control, the ratio of on time to chopper period is controlled.

① Constant Frequency System:

The another name of the system is called as pulse width modulation scheme.

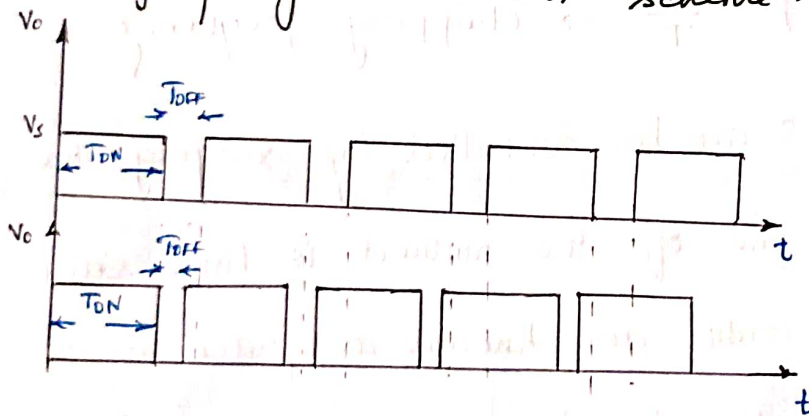
In this scheme, the on time T_{ON} is varied, but the chopping frequency f (or chopping period T) is

Kept constant, Variation of T_{ON} means adjustment of pulse width and this scheme is also called pulse width modulation scheme. Hence chopping period T is constant.

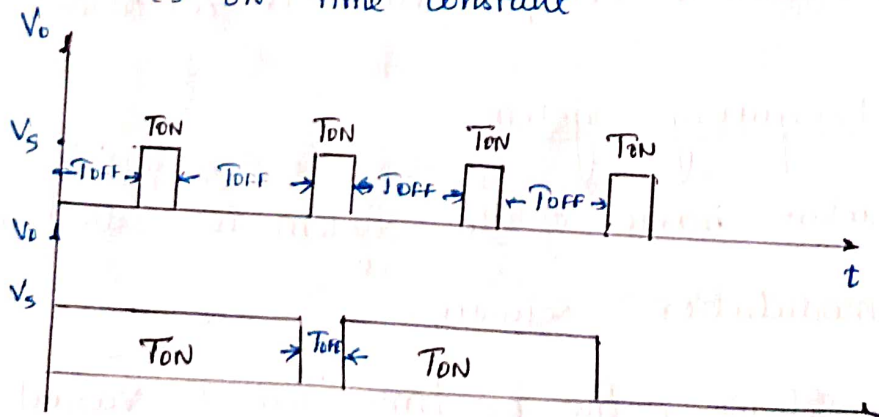


④ Variable Frequency System:

The chopping frequency (f) (or chopping period T) is varied and either (i) on time T_{ON} kept constant (ii) off time T_{OFF} is kept constant. This method of controlling α is called frequency modulation scheme.

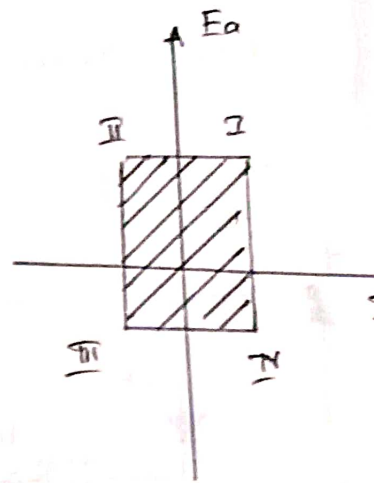
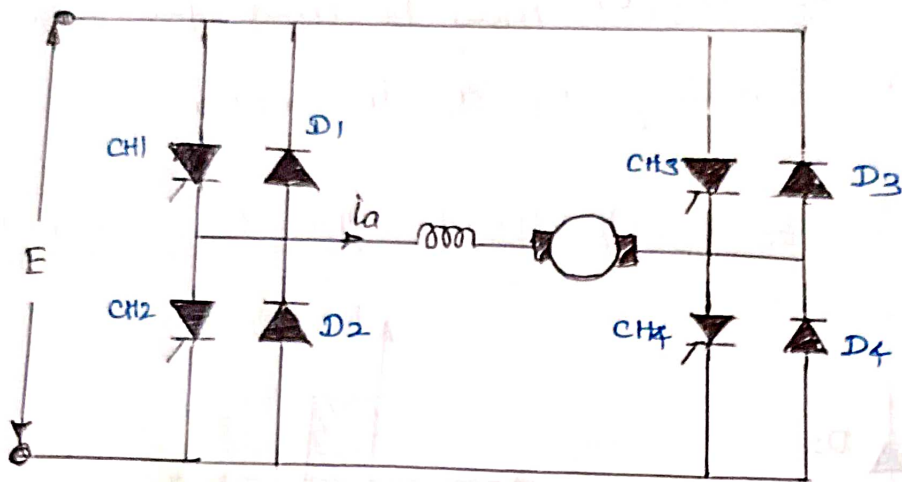


(a) ON Time constant



(b) OFF Time constant

Class - E chopper :- [Four Quadrant Chopper]



CH4 on & CH3 off , CH1 & CH2 operated
 $E_a = +ve$, I_a is reversible .

CH2 on & CH1 off , CH3 & CH4 operated
 $E_a = -ve$, I_a is reversible .

when chopper CH_1 & CH_4 are turned on, current flows through the path $E_+ - CH_1 - \text{Load} - E_-$.

Since both E_a and I_a are positive. So we get First quadrant.

* when both choppers CH_1 & CH_4 are turned off, load dissipates its energy through the path, $\text{load} - D_3 - E_+ - E_- - D_2 - \text{load}$. So E_a is negative. and I_a is positive. This operates in the fourth quadrant.

* when chopper CH_2 and CH_3 are turned on, current flows through the path, $E_+ - CH_3 - \text{load} - CH_2 - E_-$. So E_a & I_a are negative. This operates in Third quadrant.

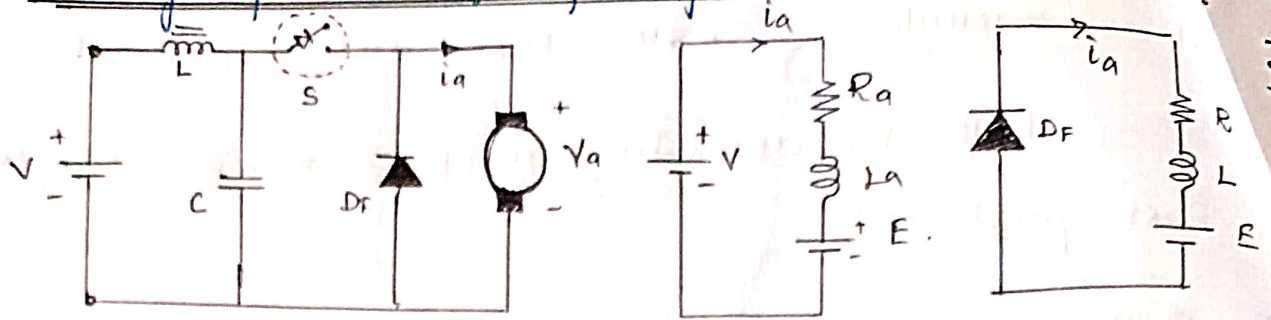
* when chopper CH_2 and CH_3 are turned off, load dissipates its energy through the path, $\text{load} - D_1 - E_+ - E_- - D_2 - \text{load}$. So E_a is positive and I_a is negative. This operates in second quadrant.

* This four quadrant choppers consists of two bridges, forward bridge & reverse bridge. chopper bridge

* CH_1 & CH_4 is forward bridge which permits energy of flow from source to load.

* CH_2 & CH_3 are reverse bridge which permits energy of flow from load to source.

Motoring operation of Separately excited Motor :-

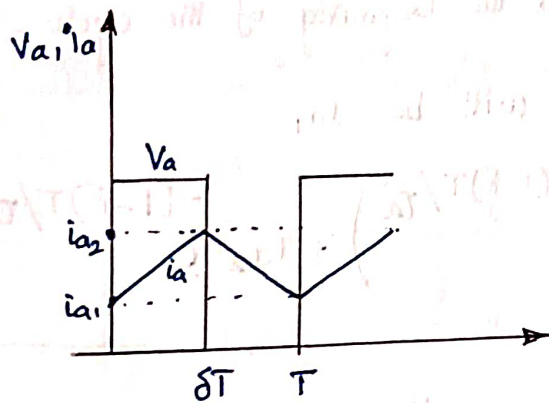


(ii) Duty interval (iii) Freewheeling interval.

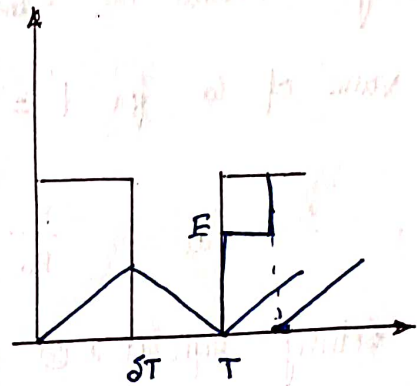
- * The LC filter is connected between the source and chopper to reduce fluctuations in the source current and voltage. When filter inductor is assumed lossless and the capacitor is sufficiently large, then the chopper input voltage will be equal to the source voltage V .
- * During the duty interval, the total energy is supplied by the source, a part of absorbed by the armature and converted into mechanical energy, a part is converted into heat in resistance R_s .
- * In this stored energy in the inductance which is responsible for maintaining the flow of armature current during the freewheeling interval, both mechanical energy and heat losses from this stored magnetic energy.
- * When the armature circuit inductance is low and the armature current is small, the stored magnetic energy may not be enough to maintain the flow of current during off period of $T_{off}(s)$, particularly when either the back emf is very large (or) duration of the off period is large.

* So the armature current is zero during freewheeling interval, giving discontinuous conduction.

* The armature current flows continuously during chopping period and the chopper is said to operate in continuous conduction during chopping period and chopper is said to operate in continuous conduction mode.



(i) continuous conduction



(ii) Discontinuous conduction

Duty Interval $(0 \leq t \leq \delta T)$.

$$R_a i_a + L_a \frac{di_a}{dt} + E = V$$

Let $i_a(0) = i_{a1}$

Apply initial condition,

$$i_a = \left(\frac{V - E}{R_a} \right) \left(1 - e^{-t/\tau_a} \right) + i_{a1} \cdot e^{-t/\tau_a}$$

where $\tau_a = \frac{L_a}{R_a}$, the armature circuit time constant.

If the current interval at the end of the duty interval

$$i_{a2} = \frac{V - E}{R_a} \left[1 - e^{-\delta T/\tau_a} \right] + i_{a1} \cdot e^{-\delta T/\tau_a}$$

Free wheeling interval ($\delta T \leq t \leq T$)

$$R_a i_a + L_a \frac{di_a}{dt} + E = 0$$

initial current at $t' = t - \delta T$

$$\therefore i_a = \frac{E}{R_a} \left(1 - e^{-t'/\tau_a} \right) + i_{a2} e^{-t'/\tau_a}$$

In the steady state value of i_a at the end of the chopping cycle should be same as at the beginning of the cycle. Thus the value of i_a for $t' = (1 - \delta)T$ will be i_{a1} .

$$\therefore i_{a1} = \frac{E}{R_a} \left(1 - e^{-(1-\delta)T/\tau_a} \right) + i_{a2} e^{-(1-\delta)T/\tau_a}$$

Solving eqn ① & ② we get,

$$i_{a1} = \frac{V}{R_a} \left[\frac{e^{\delta T/\tau_a} - 1}{e^{T/\tau_a} - 1} \right] - \frac{E}{R_a}$$

$$i_{a2} = \frac{V}{R_a} \left[\frac{1 - e^{\delta T/\tau_a}}{1 - e^{-T/\tau_a}} \right] - \frac{E}{R_a}$$

The current ripple Δi_a is

$$\begin{aligned} \Delta i_a &= \frac{i_{a2} - i_{a1}}{2} \\ &= \frac{V}{2R_a} \left[\frac{1 + e^{T/\tau_a} - e^{\delta T/\tau_a} - e^{(1-\delta)T/\tau_a}}{e^{T/\tau_a} - 1} \right] \end{aligned}$$

The steady state average value drop across the inductance is zero

$$V_a = E + I_a R_a$$

when V_a and I_a are the average value of armature terminals voltage and current

$$\delta V = E + I_a R_a \quad \therefore V_a = \delta V$$

$$\therefore I_a = \frac{\delta V - E}{R_a}$$

W.K.T Motor torque $T_a = k I_a$,

$$E = k \omega_m$$

$$\therefore \text{Speed } \omega_m = \frac{\delta V}{k} - \frac{R_a}{k^2} T_a$$

Induction Motor Drive

V/F control Method:

Advantages:

- * It provides good range of speed
- * It gives good running & transient performance
- * It has low starting current requirement
- * It has wider stable operating region.
- * Voltage & frequency reach rated value at basespeed
- * It is cheap & easy implement

Definition :-

Synchronous speed can be controlled by varying

the supply frequency.

$$N_s = \frac{120 \times f}{p} ; N_s \propto f$$

Voltage induced in the stator is

$$E_1 = 4.44 f \Phi_m T_{ph}$$

$$E_1 \propto f \Phi_m \quad \therefore 4.44 T_{ph} \rightarrow \text{constant}$$

$\Phi_m \rightarrow$ maximum airgap flux

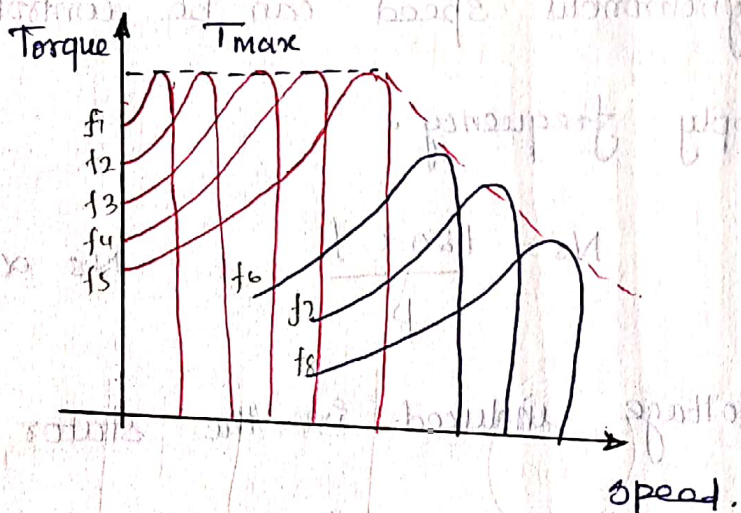
$f \rightarrow$ supply frequency.

Neglect stator voltage drop, i.e. $E_1 \approx V_1$

$V_1 \propto \Phi_m f$

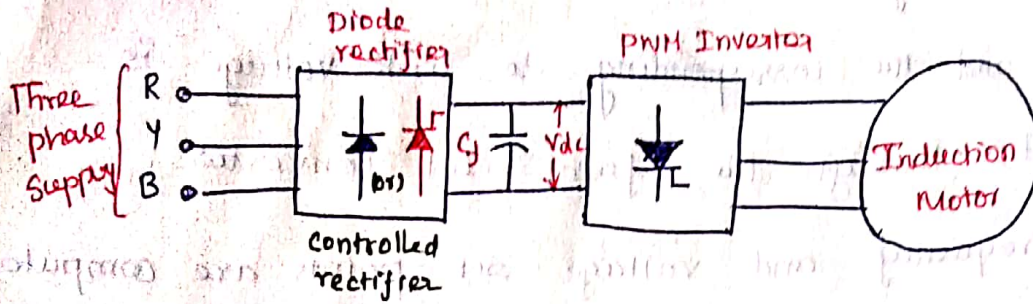
Reducing the supply frequency without changing the supply voltage will lead to increase in the airgap flux which is undesirable.

Whenever frequency is varied in order to speed control, the terminal voltage also varied so as to maintain the V/f ratio constant. Thus by maintaining a constant V/f ratio, the maximum torque of the motor becomes constant for changing speed.



The various frequency inside the operating region, the maximum torque remains the same as the speed varies.

Variable Voltage Variable Frequency supply



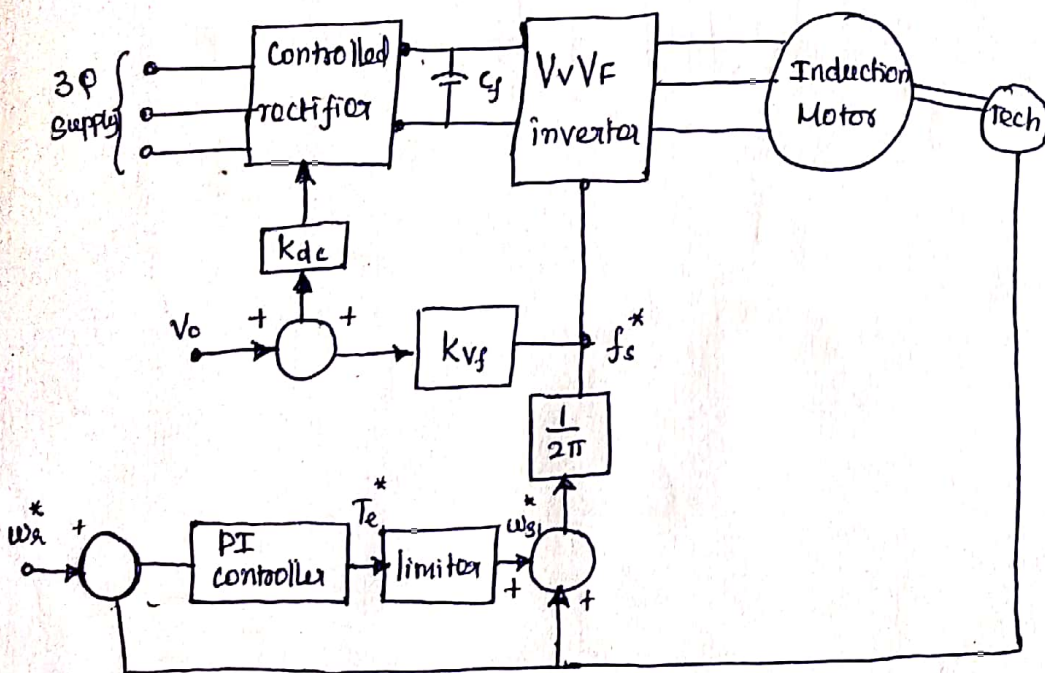
The variable voltage variable frequency supply for an induction motor drive consists of a uncontrolled or controlled rectifier and an inverter.

controlled or uncontrolled rectifier } → Fixed voltage, fixed frequency ac to variable / fixed voltage dc

inverter → dc to variable voltage / variable frequency ac

The dc link filter consists of a capacitor to keep the input voltage to the inverter stable and ripple free.

closed loop operation of V/F control :-

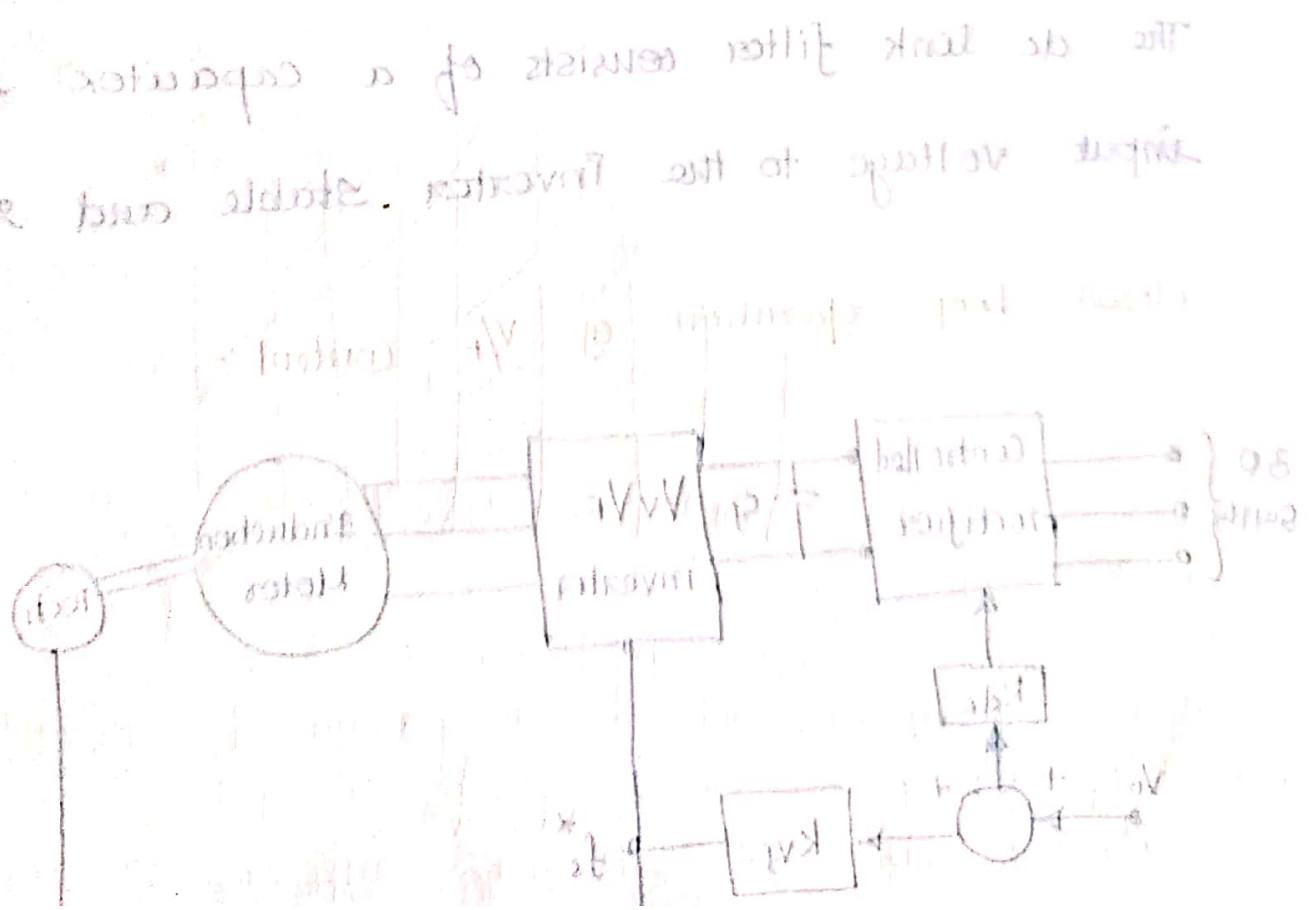


The frequency command f^* is enforced in the inverter and the corresponding dc link voltage is controlled through the front end converter.

The frequency and voltage set points are computed by PI controller loop for the inverter.

The limiter ensures that the slip speed command is added to electrical rotor speed to obtain the stator frequency command.

$K_{dc} \rightarrow$ constant proportionality between the dc link voltage and stator frequency.

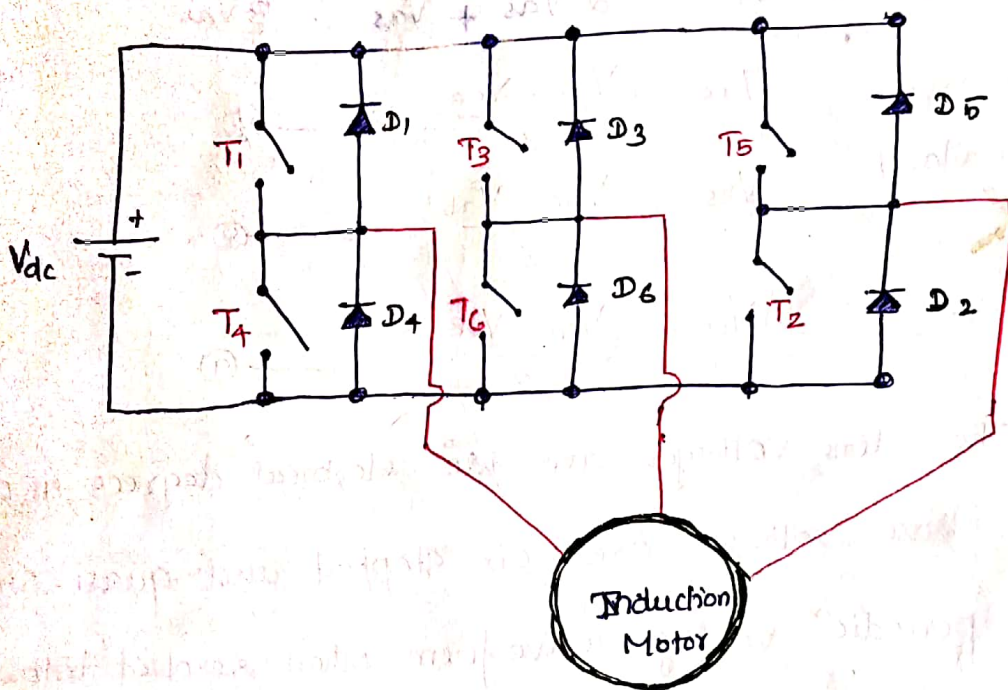


Voltage Source Inverter Driven Induction Motor

Voltage Source Inverter

Inverter is referred as a circuit that operates from a stiff DC source and generate AC output. If the input DC is a voltage source, the inverter is called as voltage source inverter. (VSI).

VSI driven Induction Motor :-



The line voltage in terms of phase voltages in a three phase system with phase sequence abc are,

$$V_{ab} = V_{as} - V_{bs} \quad \text{--- ①}$$

$$V_{bc} = V_{bs} - V_{cs} \quad \text{--- ②}$$

$$V_{ca} = V_{cs} - V_{as} \quad \text{--- ③}$$

$V_{ab}, V_{bc}, V_{ca} \rightarrow$ line voltages.

$V_{as}, V_{bs}, V_{cs} \rightarrow$ phase voltages.

then eqn ① - ③

$$V_{ab} - V_{ca} = 2V_{as} - (V_{bs} + V_{cs}) \quad \text{--- ④}$$

In balanced three phase system, the sum of three phase voltages is zero.

$$V_{as} + V_{bs} + V_{cs} = 0 \quad \text{--- ⑤}$$

$$\therefore V_{as} = - (V_{bs} + V_{cs}) \quad \text{--- ⑥}$$

Sub eqn ⑥ in eqn ④, we get

$$V_{ab} - V_{ca} = 2V_{as} + V_{as} = 3V_{as}$$

$$\therefore V_{as} = \frac{V_{ab} - V_{ca}}{3} \quad \text{--- ⑦}$$

Similarly

$$V_{bs} = \frac{V_{bc} - V_{ab}}{3} \quad \text{--- ⑧}$$

$$V_{cs} = \frac{V_{ca} - V_{bc}}{3} \quad \text{--- ⑨}$$

The line voltages are 120° electrical degrees in duration.

The phase voltages are six stepped and quasi-sine waveform

The periodic voltage waveform when resolved into fourier components have the following form.

$$V_{ab}(t) = \frac{2\sqrt{3}}{\pi} V_{dc} \left[\sin \omega t - \frac{1}{5} \sin 5\omega t + \frac{1}{7} \sin 7\omega t - \dots \right]$$

Similarly,

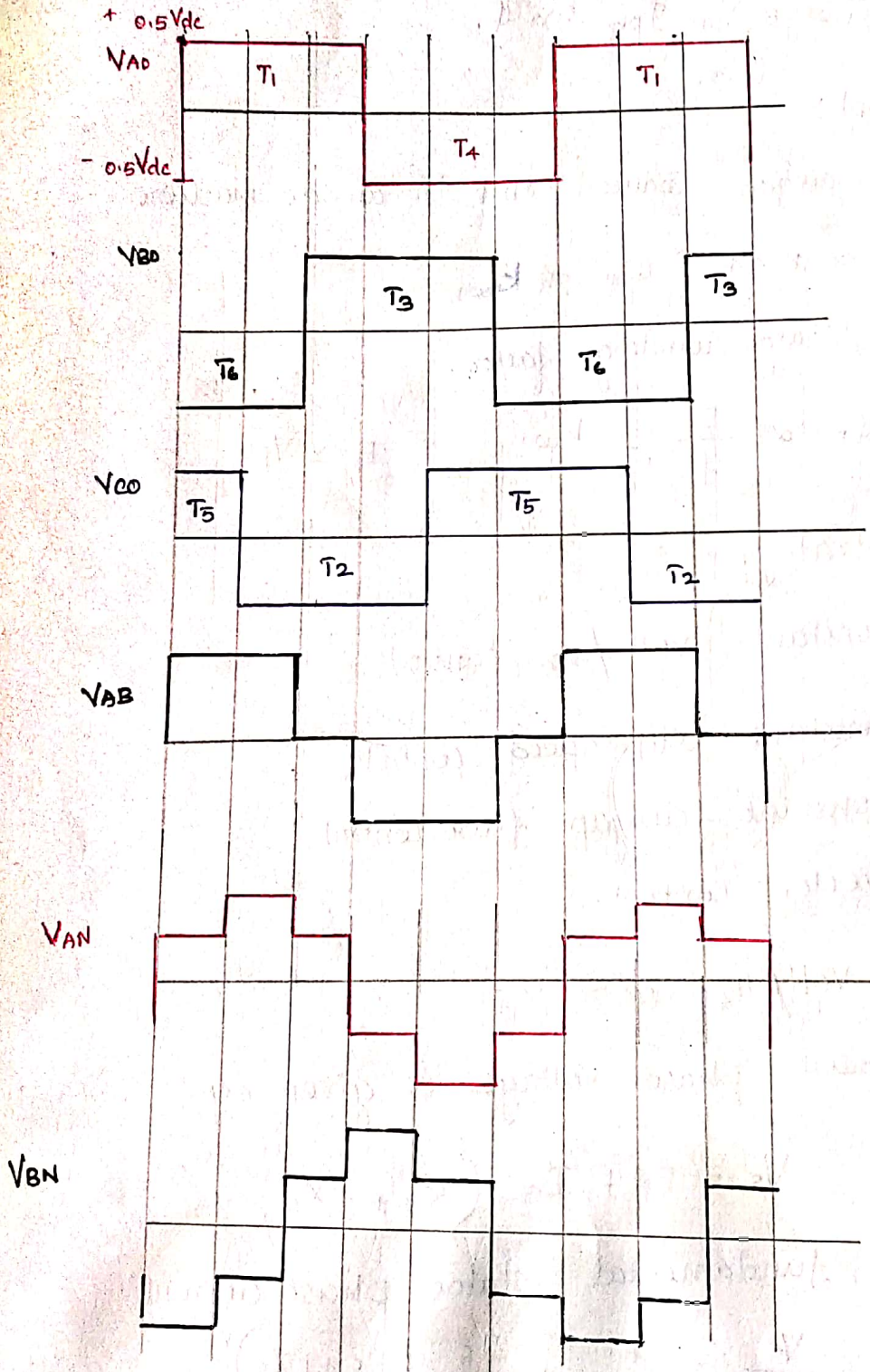
$$V_{bc}(t) = \frac{2\sqrt{3}}{\pi} V_{dc} \left\{ \sin(\omega t - 120^\circ) - \frac{1}{5} \sin(5\omega t - 120^\circ) + \frac{1}{7} \sin(7\omega t - 120^\circ) - \dots \right\}$$

$$V_{ca}(t) = \frac{2\sqrt{3}}{\pi} V_{dc} \left\{ \sin(\omega t - 240^\circ) - \frac{1}{5} \sin(5\omega t - 240^\circ) + \frac{1}{7} \sin(7\omega t - 240^\circ) - \dots \right\}$$

The fundamental rms phase voltage for the six stepped wave form is

$$V_{ph} = \frac{V_{as}}{\sqrt{2}} = \frac{2}{\pi} \times \frac{V_{dc}}{\sqrt{2}} = 0.45 V_{dc}$$

output wave form:-



Real power :-

$$P = V_{de} \times I_{de} = 3 V_{ph} I_{ph} \cos \phi,$$

$$I_{de} = 1.35 I_{ph} \cos \phi,$$

Reactive power :-

$$Q = 3 V_{ph} I_{ph} \sin \phi,$$

Speed control :-

The airgap induced emf in an ac machine

$$E = 4.44 f \Phi_m T_{ph} k_{w1}$$

$k_{w1} \rightarrow$ stator winding factor.

$$\therefore \Phi_m \propto \frac{E}{f} \propto k_{w1} \quad E \approx V_{ph}.$$

Control strategy :-

- * constant volt/Hz control
- * constant slip speed control
- * constant airgap flux control
- * Vector control.

* constant Volt/Hz control :-

The applied phase voltage is given by

$$V_s = E + I_s (R_s + jX_s)$$

$I_s \rightarrow$ fundamental stator phase current

$$\frac{V_s}{V_b} = \frac{E_1}{V_b} + \frac{I_s}{V_b} (R_s + jX_s)$$

$$V_{sn} = E_n + I_{sn} (R_{sn} + jX_{sn})$$

per unit value $V_{sn} = \frac{V_s}{V_b}$

$\therefore L_m I_m = \lambda_m$

$$E_{sn} = \frac{E_1}{V_b} = \frac{j (L_m I_m) \omega_s}{\lambda_b \omega_b}$$

$$= j \left(\frac{\lambda_m}{\lambda_b} \right) \left(\frac{\omega_s}{\omega_b} \right)$$

$$E_{sn} = j \lambda_{mn} \omega_{sn}$$

$$I_{sn} = \frac{I_s}{I_b}, \quad R_{sn} = \frac{I_b R_s}{V_b}$$

$$X_{sn} = \frac{I_b L_s \omega_s}{V_b} = L_{sn} \omega_{sn}$$

$$\lambda_{mn} = \frac{\lambda_m}{\lambda_b}$$

per unit fundamental input phase voltage is given by

$$V_{sn} = I_{sn} R_{sn} + j \omega_{sn} (\lambda_{mn} + L_{sn} I_{sn})$$

$L_{sn} \rightarrow$ stator leakage inductance (p.u)

$\omega_{sn} \rightarrow$ stator frequency (p.u)

Normalised input phase stator voltage

$$V_{sn} = \sqrt{(I_{sn} R_{sn})^2 + \omega_{sn}^2 [\lambda_{mn} + L_{sn} I_{sn}]^2} \quad (\text{p.u})$$

V/f ratio depends on,

- * frequency
- * airgap flux magnitude
- * stator impedance
- * Magnitude of stator current.

The relationship between the applied phase voltage and frequency is given by

$$V_e = V_o + k_g \cdot f_s$$

$$V_o = I_a \cdot R_s$$

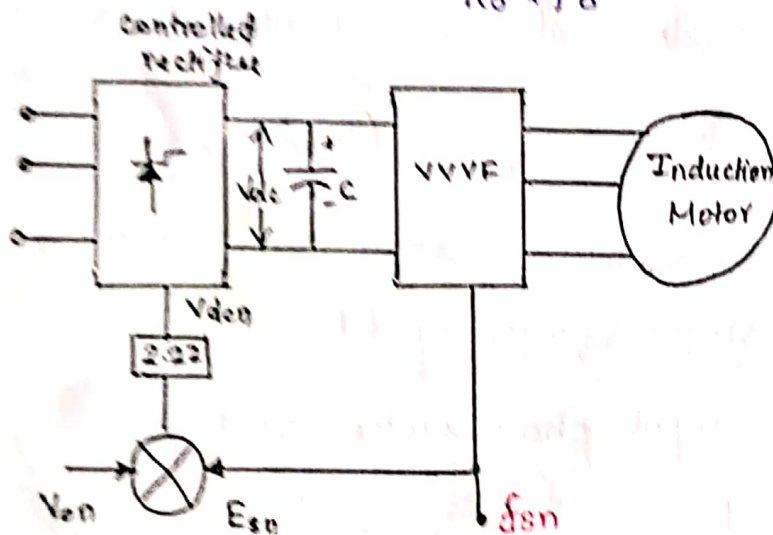
V_o → offset stator voltage to overcome the stator resistance drop.

$$V_{sn} = \frac{V_s}{V_b} = 0.45 \times \frac{V_{dc}}{V_b}$$

$$V_e = 0.45 V_{dcn}$$

$$V_{en} = \frac{V_o}{V_b} \text{ (p.u.)}$$

$$E_{in} = \frac{E_1}{b} = \frac{k_a f_s}{k_a \cdot f_b} = f_{sn}$$



$$0.45 V_{dcn} = V_{sn} + f_{sn} = V_{ob} + V_{en}$$

$$V_{dcn} = V_{em} + f_{sn} = V_{on} + E_n$$

$$V_{dcn} = 2.22 [V_{on} + f_{sn}]$$

closed loop Induction Motor Drive

It lacks

- * Slip speed
- * Offset voltage
- * Reference speed

* constant slip speed control

Slip speed of the induction motor is maintained constant, hence for various rotor speeds, the slip will be varying.

$$\omega_s = \omega_r + \omega_{se}$$

$$\omega_{se} = s \omega_s = \text{constant}$$

$$\therefore s = \frac{\omega_{se}}{\omega_s} = \frac{\omega_{se}}{\omega_r + \omega_{se}}$$

Steady state operation

The rotor current of the induction motor is given by

$$I_r = \frac{E}{\frac{R_r}{s} + jX_b} = \frac{E/\omega_s}{\left(\frac{R_r}{s} + jX_b\right)}$$

$$I_r = \frac{E/\omega_s}{\frac{R_r}{s} + jX_b} = \frac{E/\omega_s}{\frac{R_r}{s} + jX_b}$$

The electromagnetic torque is

$$T_o = \frac{P}{2} \times \frac{P_d}{\omega_s} = \frac{3 \times \frac{P}{2} \times I_r^2 R_r}{\omega_s \times s}$$

$$\therefore T_o = 3 \times \frac{P}{2} \times \frac{I_r^2 R_r}{\omega_s}$$

$$T_o = 3 \times \frac{P}{2} \times \frac{R_r/\omega_s}{\left(\frac{R_r}{\omega_s}\right)^2 + L_f^2}$$

By rearranging all the constant into one term

$$T_e = k_v \left(\frac{E_1}{\omega_s} \right)^2$$

$$\text{where } k_v = \frac{3 \times \frac{p}{2} \times \frac{R_r}{\omega_s}}{\left(\frac{R_r}{\omega_s} \right)^2 + (L_f)^2}$$

Neglect stator impedance, airgap emf = applied stator voltage

$$T_e = k_v \left(\frac{V_s}{\omega_s} \right)^2$$

* constant Airgap Flux control

constant airgap flux resolves the induction motor into an equivalent separately excited dc motor in terms of its speed of response but not in terms of the flux and the torque channel.

$$\lambda_m = L_m \cdot I_m = \frac{E_1}{\omega_s}$$

then

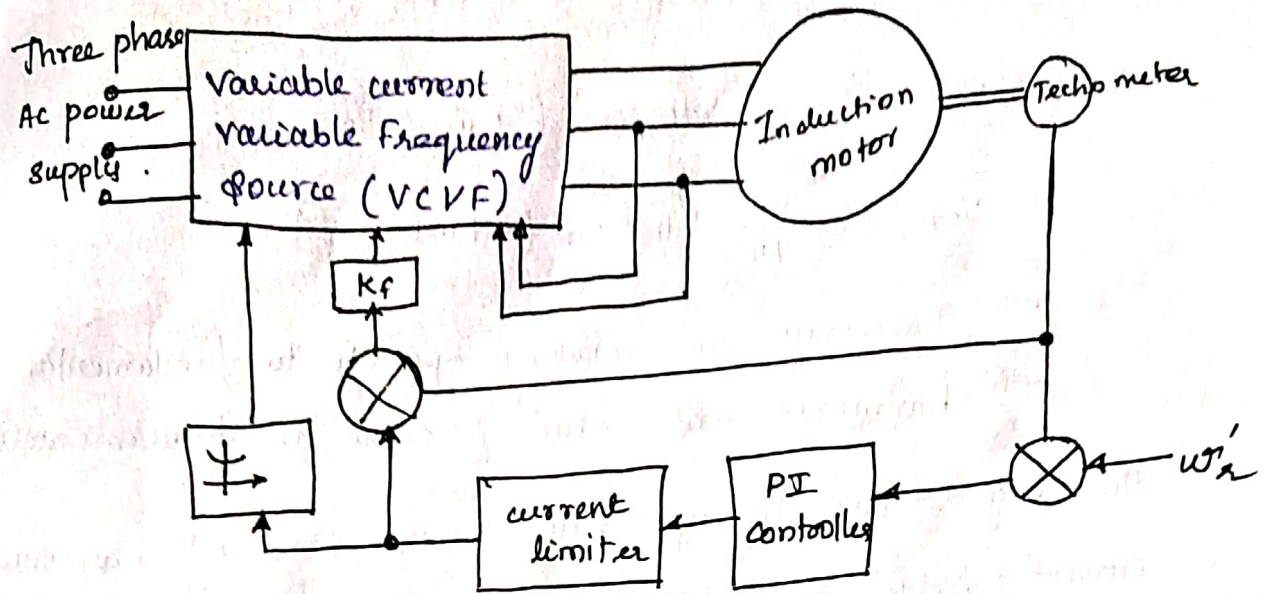
$$T_e = 3 \times \frac{p}{2} \times \lambda_m^2 \times \frac{(R_r/\omega_s)}{\left((R_r/\omega_s)^2 + (L_f)^2 \right)}$$

Assuming the airgap flux linkage is maintained constant, then the torque is

$$T_e = k_m \times \frac{R_f/\omega_s}{\left((R_f/\omega_s)^2 + (L_f)^2 \right)}$$

$$\text{where } k_m = 3 \times \frac{p}{2} \times \lambda_m^2$$

The electromagnetic torque is independent only on the slip speed.



Torque Pulsations

Six step voltage waveform generate harmonics.

The harmonics produces rotor current harmonics, which inturn interact with fundamental airgap flux, generating harmonics torque pulsations.

The harmonics torque pulsations are undesirable, they generate audible noise, speed pulsations and losses.

calculations of pulsation torque:

Using Fourier transform, (series), the voltage equations of fundamental, Fifth & Seventh harmonics

of the phase voltages are derived as,

$$V_{s1} = \frac{2}{\pi} V_{dc} \sin(\omega t - 30^\circ)$$

$$V_{s5} = \frac{2}{5\pi} V_{dc} \sin(-5\omega t - 30^\circ)$$

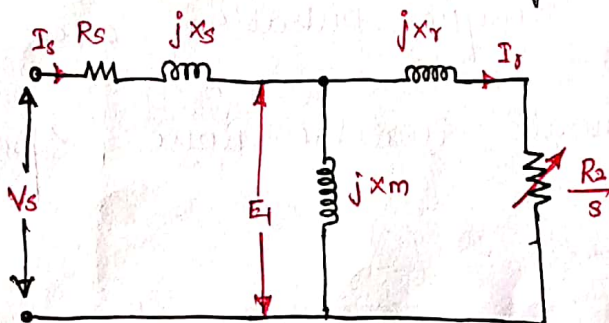
$$V_{s7} = \frac{2}{7\pi} V_{dc} \sin(+7\omega t - 30^\circ)$$

- * 5th harmonics are rotating opposite to fundamentals
- * 7th harmonics are rotating same as fundamentals.

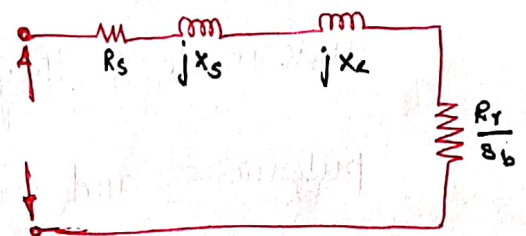
The airgap flux linkages due to the 5th & 7th harmonics current fields are revolving at six times the synchronous speed relative to the fundamental airgap flux.

Sixth harmonics torque pulsation is created by

- * The fundamental airgap flux linkages, interacting with 5th & seventh harmonic rotor current
- * The fundamental rotor current, interacting with the 5th & 7th harmonic airgap flux linkages.



Fundamental Equivalent circuit



harmonic equivalent circuit

The harmonic slip for a harmonic order h ,

$$s_b = \frac{h+1}{h} \begin{cases} + \text{ for } h \text{ odd} \\ - \text{ for } h \text{ even} \end{cases}$$

Hence $S_5 = \frac{6}{5}$, $S_7 = \frac{6}{7}$

Fundamental 5th & 7th harmonic mutual flux linkages are

$$\lambda_m = L_m I_m$$

$$\lambda_{m5} = L_m I_{m5} = \frac{I_{r5}}{5\omega_s} \left(\frac{R_r}{s_5} + j5X_r \right)$$

$$\lambda_{m7} = L_m I_{m7} = \frac{I_{r7}}{7\omega_s} \left(\frac{R_r}{s_7} + j7X_r \right)$$

and harmonic rotor currents are given by

$$I_{r5} = \frac{V_{s5}}{\left(R_s + \frac{R_r}{s_5} \right) + j5(X_s + X_r)}$$

$$I_{r7} = \frac{V_{s7}}{\left(R_s + \frac{R_r}{s_7} \right) + j7(X_s + X_r)}$$

But at frequencies above 0.3 pu, the rotor peak currents can be approximated as,

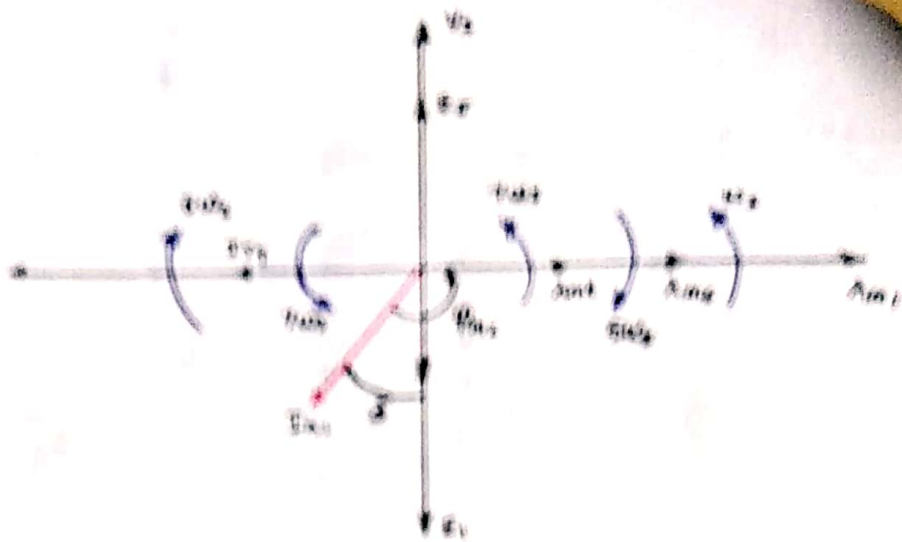
$$I_{r5} = \frac{V_{s5}}{5(X_r + X_s)} \approx \frac{2V_{dc}}{5\pi} \left(\frac{1}{5X_{eq}} \right)$$

$$\approx \frac{2V_{dc}}{\pi} \left(\frac{1}{25X_{eq}} \right)$$

and Equivalent leakage reactance is given as

$$X_{eq} = X_s + X_r$$

and $I_{r7} = \frac{2V_{dc}}{\pi} \left(\frac{1}{49X_{eq}} \right)$



$$\lambda_{m1} = I_{a1} L_e = \frac{2V_s L_e}{2\pi\omega_s} \cdot \left(\frac{L_r}{L_e} \right)$$

$$\lambda_{m1} = \frac{2V_s L_e}{4\pi\omega_s} \cdot \left(\frac{L_r}{L_{eq}} \right) \quad \text{where } L_{eq} = \frac{L_e}{\omega}$$

The fundamental torque is computed from the phasor diagram

$$T_{e1} = 3 \cdot \frac{P}{2} \cdot \lambda_{m1} I_r \sin \phi_{m1}$$

Fundamental mutual flux linkages, $\lambda_{m1} = \frac{V_s}{\omega_s} = \frac{2V_s}{\pi\omega_s}$

$$\lambda_5 = \frac{\lambda_{m1}}{25} \cdot \frac{L_r}{L_{eq}} \quad \lambda_7 = \frac{\lambda_{m1}}{49} \cdot \frac{L_r}{L_{eq}}$$

Sixth harmonic torque in anticlockwise direction.

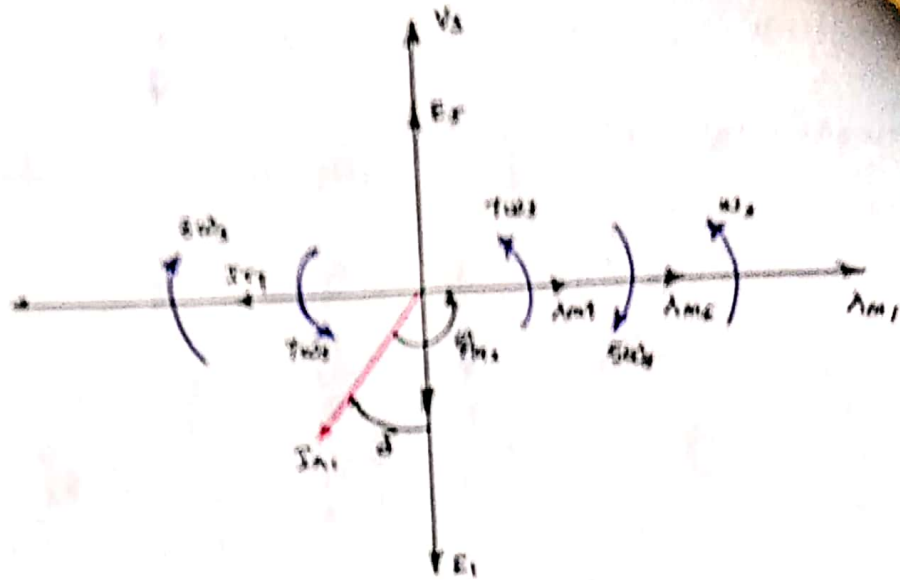
$$T_{e6} = \frac{3}{2} \cdot \frac{P}{2} \left[\lambda_{m1} (I_{r7} - I_{r5}) \sin 6\omega_s t + I_{r1} \right.$$

$$\left. \left[\lambda_{m7} \sin(6\omega_s t + 90 + \delta) + \lambda_{m5} \sin(-6\omega_s t + 90 + \delta) \right] \right]$$

$$90 + \delta = \phi_{m1}, \quad \delta \text{ become zero.}$$

$$\therefore T_{e6} = \frac{3}{2} \cdot \frac{P}{2} \left[\lambda_{m1} (I_{r7} - I_{r5}) \sin 6\omega_s t + I_{r1} (\lambda_{m7} + \lambda_{m5}) \cos 6\omega_s t \right]$$

$$\frac{T_{e6}}{T_{e1}} = \frac{I_{r7} - I_{r5}}{I_{r1}} \sin 6\omega_s t + 0.0604 \left(\frac{L_r}{L_{eq}} \right) \cos 6\omega_s t.$$



$$\lambda_{m1} = I_{r1} L_e = \frac{2V_{dc}}{25\pi\omega_s} \cdot \left(\frac{L_r}{L_e} \right)$$

$$\lambda_{m7} = \frac{2V_{dc}}{49\pi\omega_s} \cdot \left(\frac{L_r}{L_{eq}} \right) \quad \text{where } L_{eq} = \frac{X_{eq}}{\omega}$$

The fundamental torque is computed from the phasor diagram

$$T_{e1} = 3 \times \frac{P}{2} \times \lambda_{m1} I_r \sin \phi_{m1}$$

Fundamental mutual flux linkages, $\lambda_{m1} = \frac{V_s}{\omega_s} = \frac{2V_{dc}}{\pi\omega_s}$

$$\therefore \lambda_{\sigma} = \frac{\lambda_{m1}}{25} \times \frac{L_r}{L_{eq}} \quad , \quad \lambda_{\tau} = \frac{\lambda_{m1}}{49} \times \frac{L_r}{L_{eq}}$$

Sixth harmonic torque in anticlockwise direction,

$$T_{e6} = \frac{3}{2} \times \frac{P}{2} \left[\lambda_{m1} (I_{r7} - I_{r5}) \sin 6\omega_s t + I_{r1} \left[\lambda_{m7} \sin(6\omega_s t + 90 + \delta) + \lambda_{m5} \sin(-6\omega_s t + 90 + \delta) \right] \right]$$

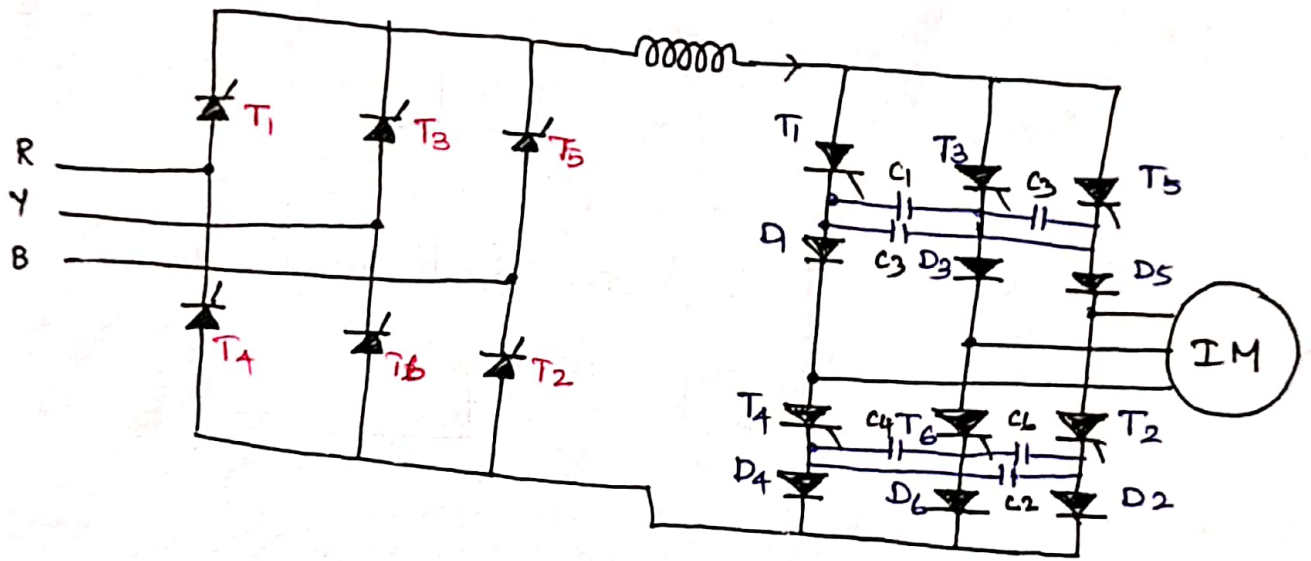
$$90 + \delta = \phi_{m1} \quad , \quad \delta \text{ become zero.}$$

$$\therefore T_{e6} = \frac{3}{2} \times \frac{P}{2} \left[\lambda_{m1} (I_{r7} - I_{r5}) \sin 6\omega_s t + I_{r1} (\lambda_{m7} + \lambda_{m5}) \cos 6\omega_s t \right]$$

$$\frac{T_{e6}}{T_{e1}} = \frac{I_{r7} - I_{r5}}{I_{r1}} \sin 6\omega_s t + 0.0604 \left(\frac{L_r}{L_{eq}} \right) \cos 6\omega_s t.$$

Current Source Inverter

In a current source drive, the input currents are six stepped waveforms. Amplitude and frequency are variable.



The converter system has a controlled rectifier for providing the ac-to-dc conversion and an inverter for dc-to-ac conversion.

The dc output voltage is fed to the auto-sequentially commutated current source inverter through a filter inductor. This inductor is provided to maintain the dc link current at a steady value.

Commutation:-

The sequence of firing the ASCR is $T_1, T_2, T_3, T_6, T_5, T_4$ for the phase sequence abc in the induction motor. At any time two SCR's are conducting, they turn on at the interval of 60° .

Speed control of Induction Motor

A three phase induction motor is practically a constant speed motor. Speed control is achieved by power factor, efficiency etc.

$$N = N_s (1-s)$$

$$T \propto \frac{s E_2^2 R_2}{R_2^2 + (sX_2)^2}$$

The speed of the motor can be controlled by,

— From stator side

- * Supply frequency control to control N_s (V/f)
- * Supply voltage control
- * Controlling no of stator poles to control N_s
- * Adding rheostat in stator.

— From rotor side

- * Adding external resistance in the rotor circuit
- * cascade control
- * Injecting slip frequency voltage into the rotor circuit.

Supply Frequency control or V/f control

$$N_s = \frac{120f}{P}$$

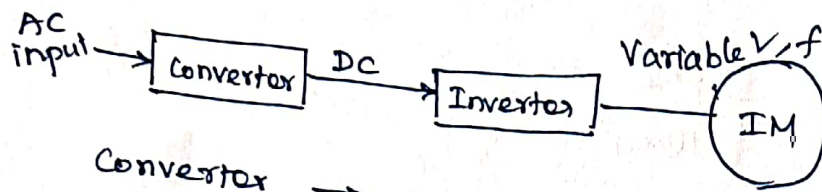
and $E = 4.44 f \Phi_m K_w T_{ph}$

$$\Phi_m = \frac{E}{4.44 f K_w T_{ph}}$$

$$\Phi_m = \frac{1}{4.44 K_w T_{ph}} \times \left(\frac{E}{f} \right)$$

In general, $\phi \propto \frac{V}{f}$

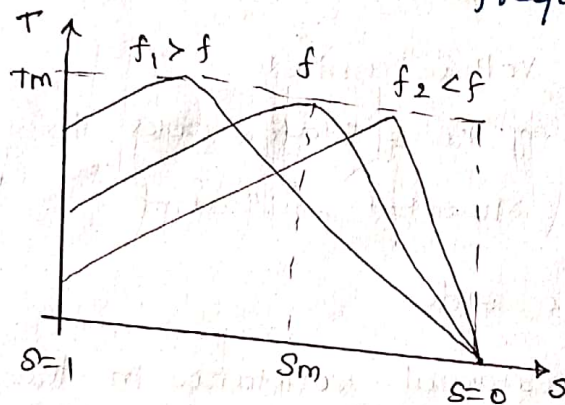
The supply frequency f changed, the value of air gap flux ϕ get affected. This may result into saturation of stator and rotor core.



Converter \rightarrow Converts AC supply to DC supply

Inverter \rightarrow Converts DC supply to AC supply.

The inverter output get variable frequency and variable supply.

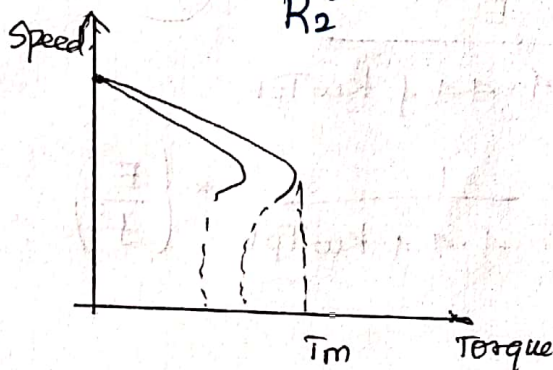


Supply voltage control:

$$T \propto \frac{s E_2^2 R_2}{R_2^2 + (s X_2)^2}$$

E_2 , the rotor induced emf at standstill depends on the supply voltage V . $E_2 \propto V$ and $(s X_2)^2 \ll R_2^2$

$$\therefore T \propto \frac{s V^2 R_2}{R_2^2} \propto s V^2 \text{ for constant } R_2.$$



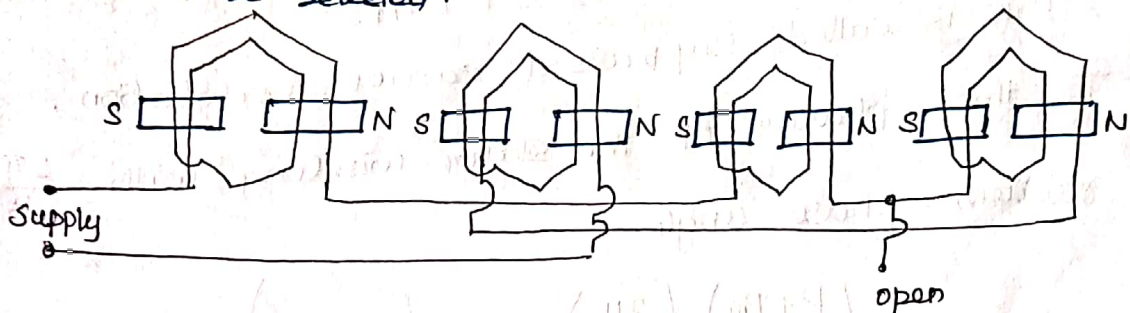
controlling of No of poles.

This method is called pole changing method of controlling the speed. The stator poles can be changed by

- 1) Consequent poles method
- 2) Multiple stator winding method
- 3) Pole amplitude modulation method.

Consequent Pole method :-

The connection of stator winding are changed with the help of simple switching. Due to this, the no of stator poles get changed in the ratio 2:1. Hence two synchronous speed can be selected.



Multiple stator winding method :-

In this method, instead of one winding, two separate stator windings are placed in the stator core. The windings are placed in the stator slots only but are electrically isolated from each other. Each winding is divided into coils to which, pole changing with consequent poles, facility is provided.

Limitations :-

- * Only applicable in squirrel cage motor.
- * smooth speed control is not possible.
- * Increases the cost of the motor.
- * complicated to design the motor.

Pole Amplitude Modulation:

The basic principle of this method is the modulation of two sinusoidally varying mmf waves, with different no of poles.

$$f(\theta) = F \sin\left(\frac{P}{2}\theta\right) \quad \theta \rightarrow \text{mechanical angle.}$$

$$f_m(\theta) = M \sin\left(\frac{P_m}{2}\theta\right)$$

$$f_R(\theta) = FM \sin\left(\frac{P}{2}\theta\right) \sin\left(\frac{P_m}{2}\theta\right)$$

$$= \frac{1}{2} FM \left[\cos\left(\frac{P-P_m}{2}\theta\right) - \cos\left(\frac{P+P_m}{2}\theta\right) \right]$$

$$P_1 = P - P_m, \quad P_2 = P + P_m.$$

This is called suppressed carrier modulation.

The three phases of the stator winding with $\frac{2\pi}{3}$ radian phase angle.

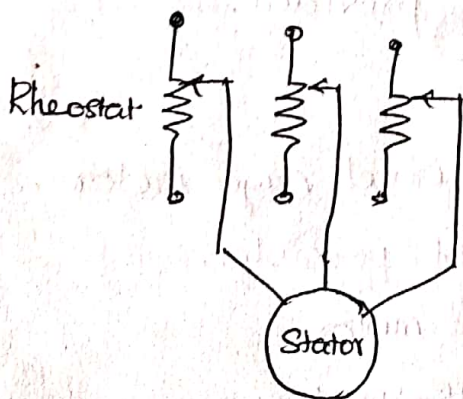
$$\left(\frac{P+P_m}{2}\right) \left(\frac{2\pi}{3}\right) \lambda = \left(1 + \frac{P_m}{P}\right) \left(\frac{2\pi}{3}\right) \lambda$$

$$\left(1 + \frac{P_m}{P}\right) \left(\frac{2\pi}{3}\right) \lambda = 2\pi n$$

$$1 + \frac{P_m}{P} = \frac{3n}{\lambda}$$

$$\frac{n}{\lambda} = \frac{1}{3} \left[1 + \frac{P_m}{P}\right]$$

Adding External Resistance in stator or rotor circuit



The reduction in stator voltage cause reduction in speed of the motor.

External Resistance in Rotor.

$$T \propto \frac{s E_2^2 R_2}{R_2^2 + (sX_2)^2}$$
$$(sX_2)^2 \ll R_2.$$

then $T \propto \frac{s}{R_2}$.

If the rotor resistance increased, the torque produced decreases. But when the load on motor is same.

Disadvantages.

- * Large speed changes are not possible
- * It can not be used in squirrel cage induction motor
- * Large power loss occurs.
- * Due to power loss, efficiency is low.

Cascaded Control

In this method, two induction motor mounted on same shaft. Main motor \rightarrow slip ring induction motor.

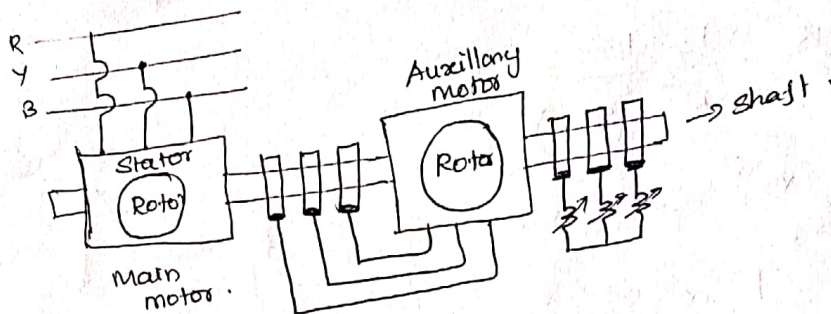
Auxiliary motor \rightarrow another slip ring induction motor.

The supply of auxiliary motor derived from main motor at slip frequency from the slip ring induction motor.

This is called cascading of the motor.

If the torque produced by both act in same direction, cascading is called cumulative cascading. If torque produced are in opposite direction, cascading is called differential cascading.

$$N = \frac{120f}{P_A + P_B}$$



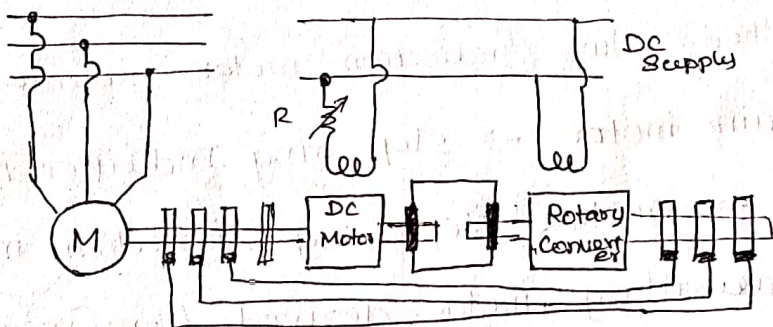
Injecting slip frequency Emf into Rotor circuit!

In this method, the voltages are injected in the rotor circuit. The frequency of the rotor circuit is a slip frequency and hence the voltage to be injected must be at the slip frequency.

Two methods available,

- 1) Kramer system
- 2) Scherbius system

Kramer System:

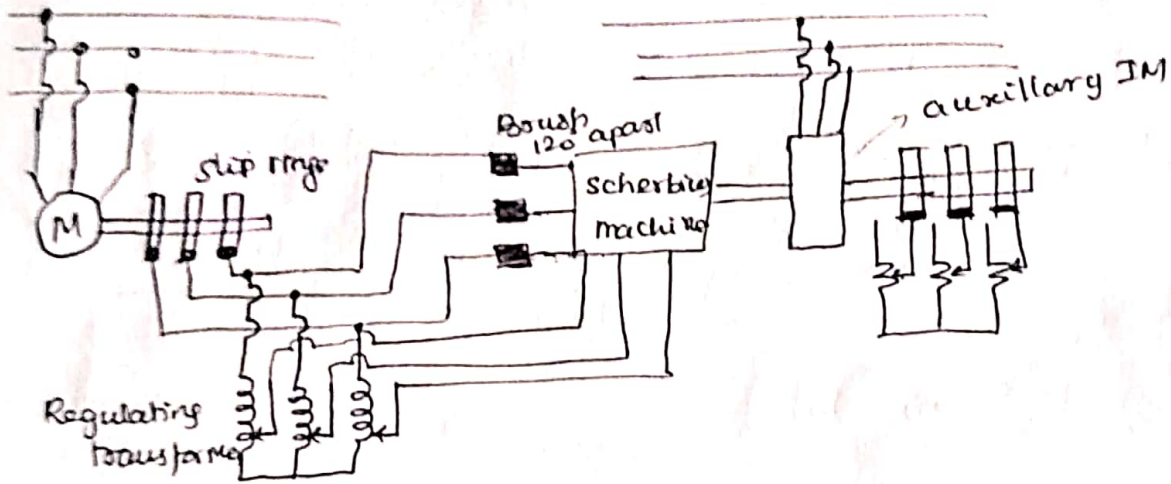


It consists of main Induction Motor M , the speed of which is to be controlled, The two additional equipments are DC motor & rotary converter. The slip ring are connected to ac side of rotary converter. DC shunt motor connected to DC side of the rotary converter.

The field ~~resistor~~ rheostat can be varied by changing the field supply to the DC motor. The change of DC voltage at rotary converter side.

Variable frequency converter has fixed ratio between its a.c side and d.c side voltages. Thus voltage on its ac side also changes. This ac voltage is given to the slip ring of the main motor. So the voltage injected in the rotor of main motor changes which produces the required speed control.

Scherbius System:



This method requires an auxiliary 3 phase or 6 phase ac commutator machine is called scherbius machine. This system is not directly connected to the main motor, whose speed is to be controlled.

Scherbius machine excited by slip frequency from the rotor of main motor through a regulating transformer. The various voltage at the scherbius system, which is injected into the rotor of main motor. This control speed of the main motor.

Synchronous Motor Drive

Variable Frequency Control

Synchronous speed is directly proportional to frequency of the input voltage.

$$N_s = \frac{120f}{P} ; N_s \propto f$$

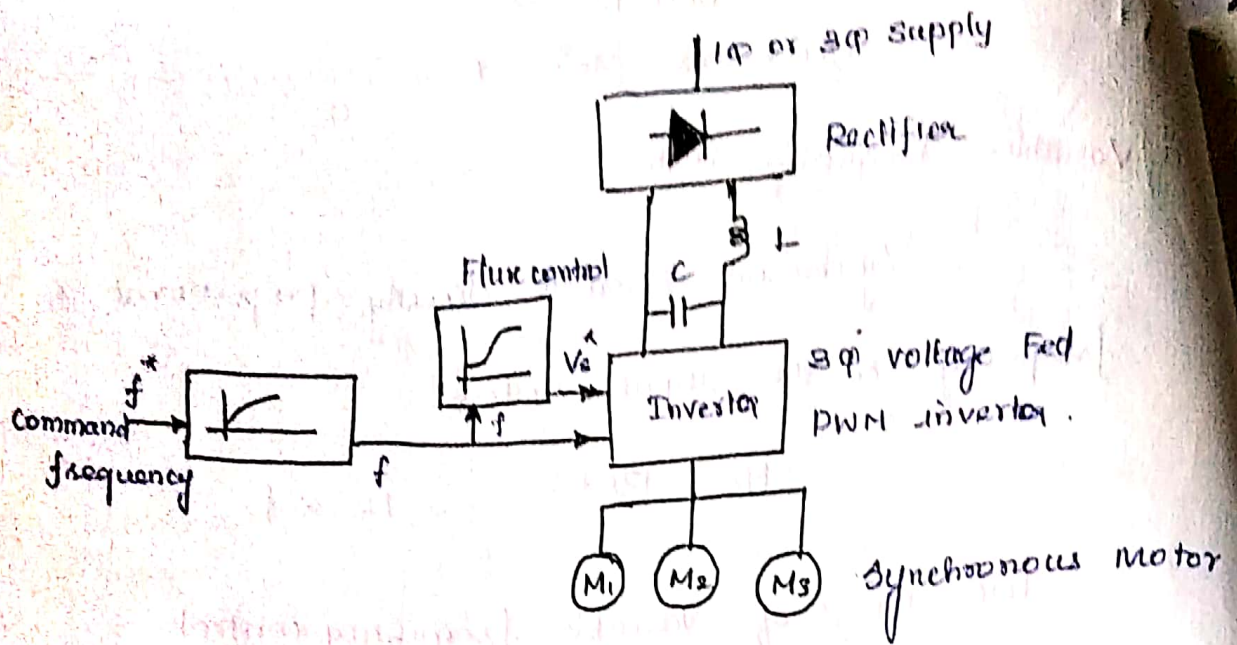
Two modes of variable frequency control

- 1) True synchronous mode (or) separate control mode
- 2) Self controlled mode.

Separate controlled Mode:

True synchronous mode (or) separate control mode of the synchronous drives are having separate controlled of gate signal. The controlling signals are generated externally to the drives.

This method used for smooth starting and regenerative braking. We can explain the true synchronous mode using v/f control method (open loop)



All the machines are connected in parallel to the same inverter and they move in response to the command frequency f^* at the input.

The frequency f^* after passing the delay circuits is applied to the voltage source inverter (or) voltage fed PWM inverter. This is done so that rotor source able to track the change in frequency.

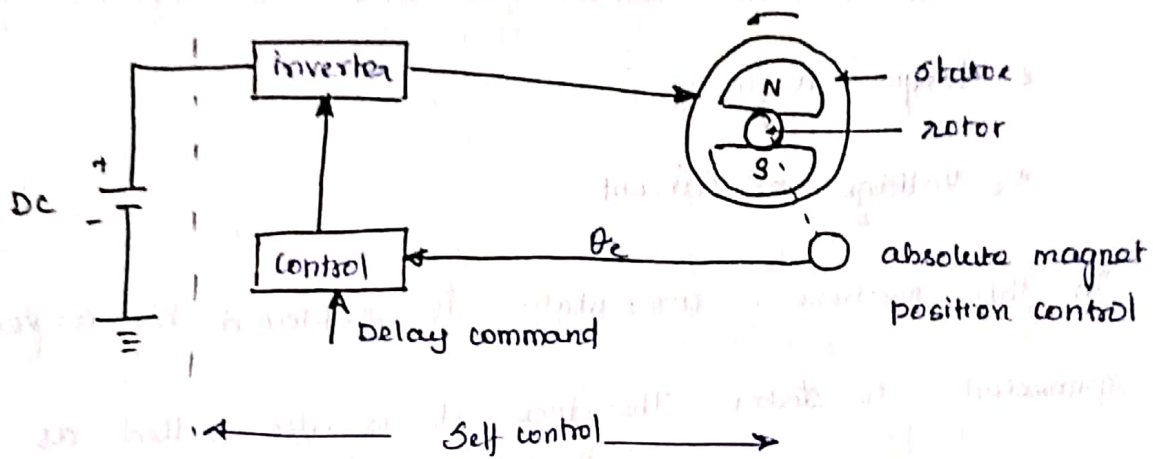
The flux control block is used which changes the stator voltage with frequency so as to maintain constant flux for speed below base speed and constant terminal voltage for speed above base speed.

below base speed \rightarrow constant flux. (V/f).

above base speed \rightarrow constant voltage

The front end of the voltage fed PWM inverter is supplied from utility line through a diode rectifier and LC filter. The machine can be built with damper winding to prevent oscillations.

Self controlled Mode:



In self control mode, the supply frequency is changed so that the synchronous speed is same as that of the rotor speed. Hence rotor cannot pull out of slip and hunting eliminations are eliminated. For such a mode of operation, the motor does not require a damper winding.

The stator winding of the machine is fed by an inverter that generates a variable frequency voltage (sinusoidal) supply.

The frequency and phase of the output wave are controlled by an absolute position sensor mounted on machine shaft, giving it self control characteristic. Here the pulse train from position sensor may be delayed by external command.

The machine control behavior is decided by

- * torque angle
- * Voltage or current.

In this machine, commutator is replaced by converter connected to stator. Therefore it is also called as commutator less motor (CLM).

The firing pulses are obtained from phase of stator voltages. When synchronous motor is over excited they can supply reactive power required for commutation thyristors. Now the inverter works as line commutated inverter where the firing signals are synchronized with line voltages.

The frequency of the inverter will be same as that of the machine voltages. (synchronized). This type of inverters are called load commutated inverter (LCI).

Comparison between Self and separately SM Drive.

seperately control	self control
1) Hunting oscillations are present	Hunting is eliminated
2) Damper winding is required	No need for damper winding.
3) Stator supply frequency is controlled from an independent oscillator	No need for frequency adjustment from independent oscillator.
4) Multiple no. of machine can be controlled	Single machine is controlled.

Power Factor Control

Definition:

In linear loads, power factor = $\cos \phi$

$\phi \rightarrow$ angle between voltage and current of the phase.

In non linear load is fed from a sinusoidal supply. Current will consists of fundamental and harmonics.

$$\text{Real power} = 3 V I \cos \phi$$

$$\text{reactive power} = 3 V I \sin \phi$$

$$\text{Apparent power} = 3 V I$$

$$\text{power factor} = \frac{\text{Real power}}{\text{Apparent power}} = \frac{3 V I \cos \phi}{3 V I_{\text{rms}}}$$

$$\text{power factor} = \cos \phi \times \left(\frac{I}{I_{\text{rms}}} \right)$$

$$= \text{displacement factor} \times$$

$$\text{Distortion factor.}$$

$$\text{Displacement factor} = \cos \phi$$

$$\text{Distortion factor} = \frac{I}{I_{\text{rms}}}$$

Drives operation on low power factor :- (Applications)

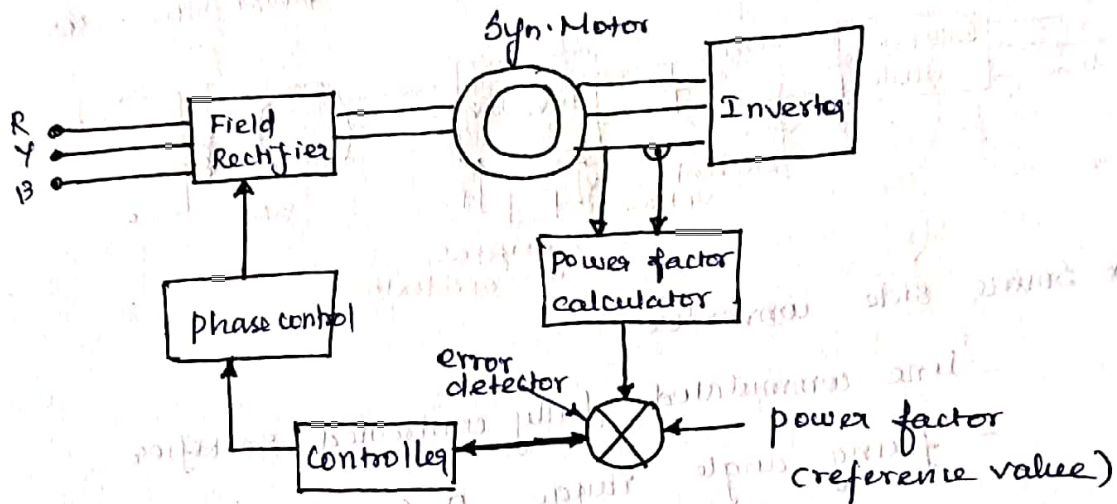
- * Ac induction motor direct on line.
- * Ac - Dc diode rectifier
- * line commutated thyristor fed dc motor
- * Variable frequency Ac motor drive.
- * Ac regulator fed induction motor drive
- * Induction motor drive with slip power recovery

Benefits of power factor control :-

- * Power factor increases tends to reduce copper loss.

- * It helps in stabilizing the system voltage
- * It reduces the load on transmission & distribution side.
- * It avoid large penalty often imposed on low powerfactor.

Power Factor control:

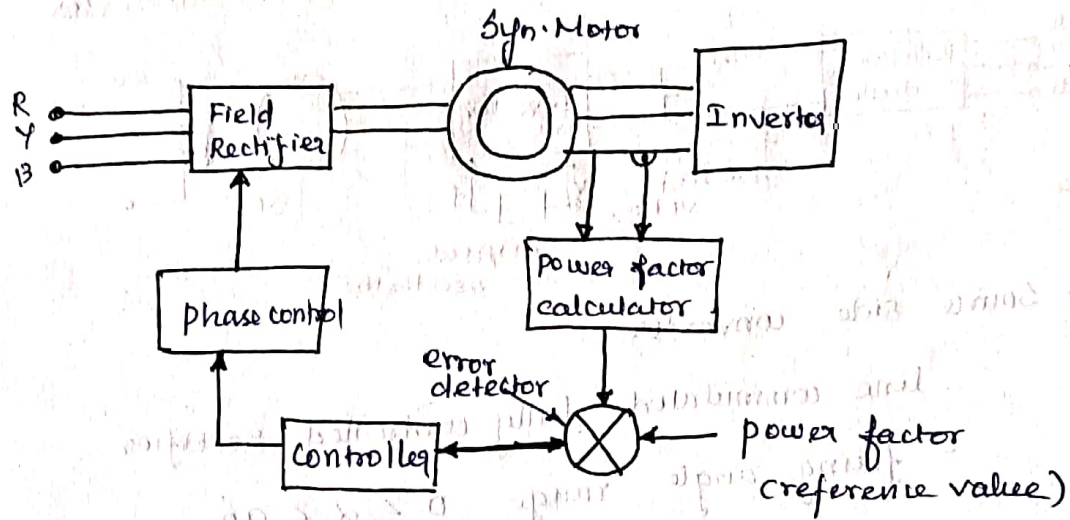


The main aim of adjustment of power factor is the variation of field current. The motor voltage and currents are sensed and fed to the power factor calculator. The p.f calculator computes the phase angle between the two (voltage & current). It is actual power factor value.

The actual p.f is compared with reference power factor by using error detector. The error is amplified by error amplifier and its output varies the field current until p.f confirm to the reference value.

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- * It reduces the load on transmission & distribution side.
- * It avoid large penalty often imposed on low power factor.

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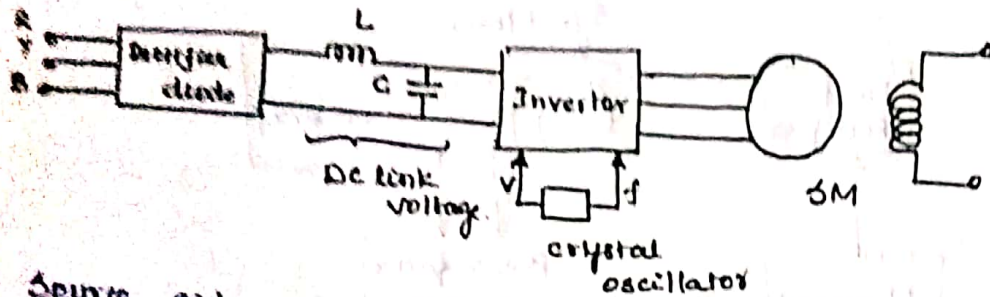
The actual p.f is compared with reference power factor by using error detector. The error is amplified by error amplifier and its output varies the field current until p.f confirm to the reference value.

Self controlled synchronous motor Drive employing LCC

In synchronous motor drive consists of two converters

- * Source side converter
- * Load side converter.

operation control of SM fed from PWM inverter.



* Source side converter

- line commutated fully controlled rectifier
- firing angle range $0 \leq \alpha \leq 90^\circ$
- V_{ds} , i_{ds} are ~~positive~~ positive.

* Load side converter

- firing angle range $90^\circ \leq \alpha \leq 180^\circ$
- line commutated fully controlled inverter.
- V_{ds} is negative, i_{ds} is positive.

Leading power factor operation

- Load side 3 ϕ converter commutated by motor induced voltages.

- Source side 3 ϕ converter commutated by supply voltage

Self control of SM fed square wave inverter

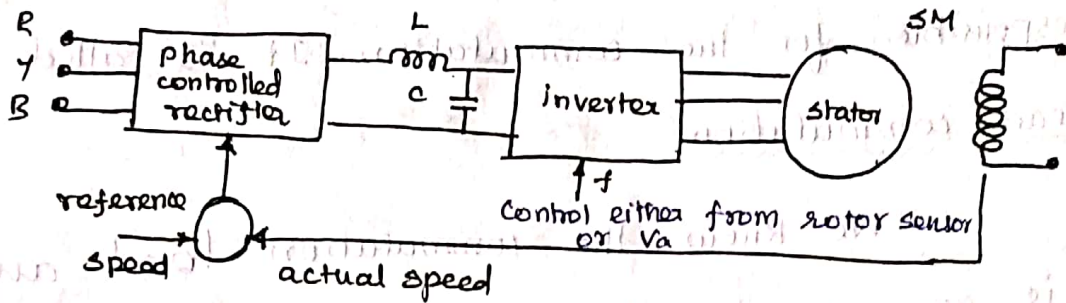
* Motoring operation

* Firing angle range $0 \leq \alpha \leq 90^\circ$

* ~~Source side~~ source side converter acting as rectifier.

* Load side converter acting as inverter with firing angle $90 \leq \alpha \leq 180$

* Power flow from source to load. This operation is called Motoring operation.



* Regenerative Braking

* Firing angle range $0 \leq \alpha \leq 90$, load side converter acting as a rectifier.

* Firing angle range $90 \leq \alpha \leq 180$, source side converter acting as a inverter.

* power flow from load side to source side

This operation is called as regenerative braking.

Now the V_{ds} is negative, i_{ds} is positive.

To avoid commutation overlap, the commutation lead angle for load side converter is

$$\beta = 180 - \alpha$$

* Input ac current behind the supply voltage by angle α .

Constant Marginal Angle Contact

In a converter, if the commutation is done by line voltages, then it is called as line commutation. But if the voltage induced in the load are responsible for the commutation. It is called as load commutation.

We know the commutation load angle β is given by

$$\beta = 180 - \alpha \quad \text{--- ①}$$

The marginal angle γ is given by

$$\gamma = \beta - \mu \quad \text{--- ②}$$

and for the safe commutation of thyristor of load commutation CSI

$$\gamma > \omega t_q \quad \text{--- ③}$$

$t_q \rightarrow$ turn off time of thyristor.

The power factor angle of synchronous motor is given by

$$\phi = \beta - 0.5 \mu \quad (\text{leading}) \quad \text{--- ④}$$

In a motoring operation,

- * power factor is maximum
- * β is minimum (or) α is maximum.
- * power transferred from dc link to machine.
- * It gives i_d is maximized in turn maximize the machine torque.

∴ A minimum value of β is obtained when the margin angle is chosen just sufficient. i.e. γ_{min} to ensure safe commutation.

$$\therefore \gamma_{min} = K_g \omega t_g \quad \text{--- (5)}$$

$K_g \rightarrow$ safety factor.

Now minimum value of β is

$$\beta_{min} = \mu + \gamma_{min} \quad \text{--- (6)}$$

For maximum power factor, $(PF)_{max} = \cos(\beta_{min} - 0.5\mu)$

$$(PF)_{max} = \cos(\gamma_{min} + 0.5\mu) \quad \text{--- (7)}$$

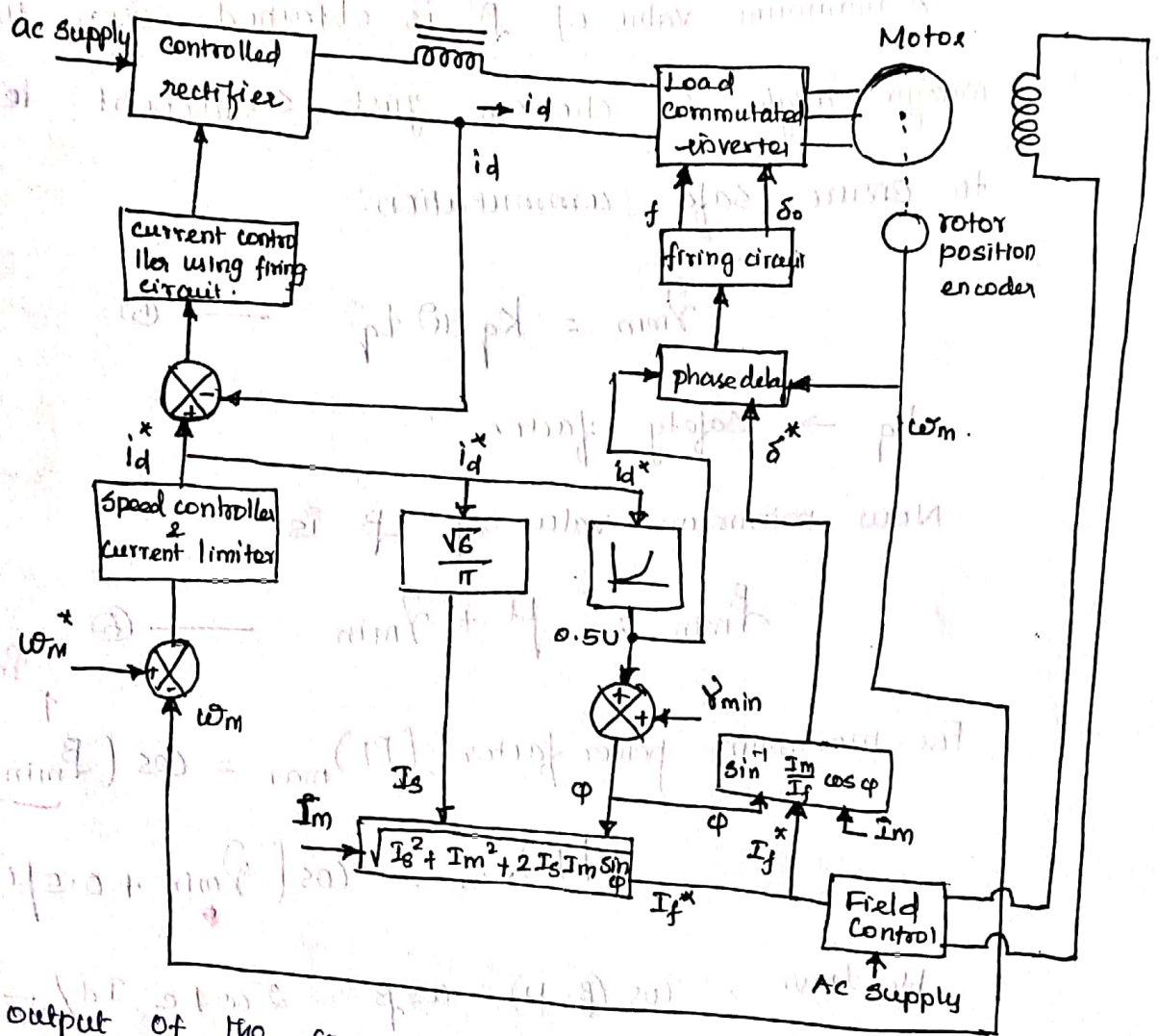
$$\text{We have, } \cos(\beta - \mu) - \cos\beta = 2\omega L_c I_d / \sqrt{6} V \quad \text{--- (8)}$$

From eqn (6) & (7), we get

$$\cos(\mu + \gamma_{min}) = \cos\gamma_{min} - 2\omega L_c I_d / \sqrt{6} V \quad \text{--- (9)}$$

Constant Marginal Angle control closed loop operation

The constant marginal angle control for wound field motor drive employing a rotor position encoder. This drive has an outer speed loop and an inner current loop. The rotor current can be sensed by using rotor position control encoder. It gives actual speed ω_m . The comparator compares ω_m & ω_m^* (reference value).



The output of the comparator is fed to the speed controller and current controller. It gives reference current value i_d^* . I_d is sensed by current sensor and fed to the comparator.

The output of the comparator is fed to the current controller. It generates the trigger pulses.

Load commutated drives used for medium, high and very high power drives. High speed drives such as compressors, extractors, induced and forced draft fans, blowers, conveyors, aircraft test facilities, steel rolling mills, large ship propulsion, main line traction, flywheel energy storage.

Permanent Magnet Synchronous Motor Drives.

PMSM are commonly known as Permanent Magnet ac motors (PMAC). It is classified

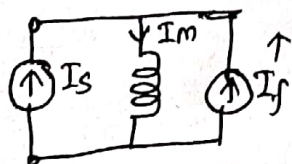
- * Sinusoidal PMAC \rightarrow induced voltage in sinusoidal
- * Trapezoidal PMAC \rightarrow induced voltage in trapezoidal.

Sinusoidal PMAC Motor drives.

A Sinusoidal PMAC motor has distributed winding in the stator side. It employs rotor geometries such as inset or interior. Rotor poles are so shaped that the voltage induced in stator phase winding has a sinusoidal waveform.

Since the voltages produced in the stator of sinusoidal PMAC motor are sinusoidal, ideally, the three stator must be supplied with variable frequency sinusoidal voltages (or) currents with phase difference of 120° between them.

Equivalent circuit:



$$I_f = \frac{V}{jX_s} + \frac{V}{R_s} \angle - (\delta + \theta_s)$$

$$I_m = I_s + I_f$$

The mechanical power developed by the motor is

$$P_m = 3 X_s I_s I_f \sin \delta'$$

$$T = \frac{P_m}{\omega_s} = k I_s I_f \sin \delta'$$

where $k = \frac{3 X_s}{\omega_s}$

For $\delta' = \pm 90^\circ$, then $T = \pm k I_f I_s$

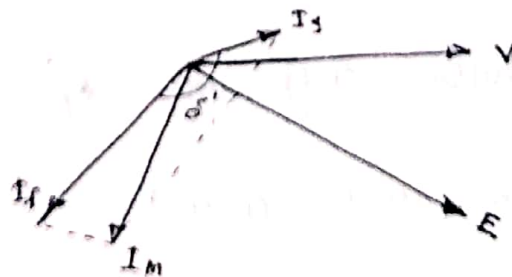
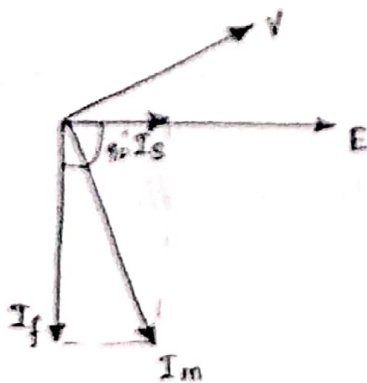
$$T = \pm k_T I_s$$

$$k_T = k I_f$$

Here Torque is directly proportional to I_s .

The maximum torque obtained by $\delta' = 90^\circ$

In this condition, motor is said to operate with unity internal power factor because I_s is in phase with excitation emf E . (Motoring operation)



$$I_f = \frac{E}{jX_s} = \frac{E}{X_s} \angle -(\delta + \pi/2)$$

$$I_m = I_s + I_f$$

The mechanical power developed by the motor is

$$P_m = 3 X_s I_s I_f \sin \delta'$$

$$T = \frac{P_m}{\omega_s} = k I_s I_f \sin \delta'$$

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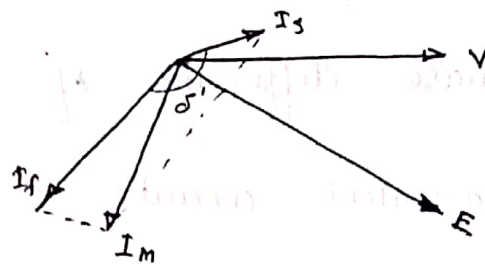
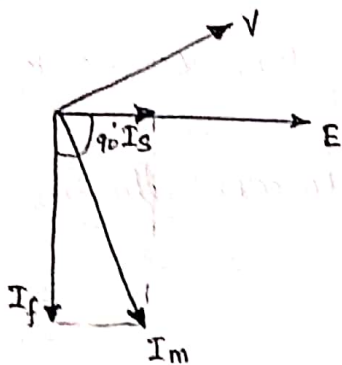
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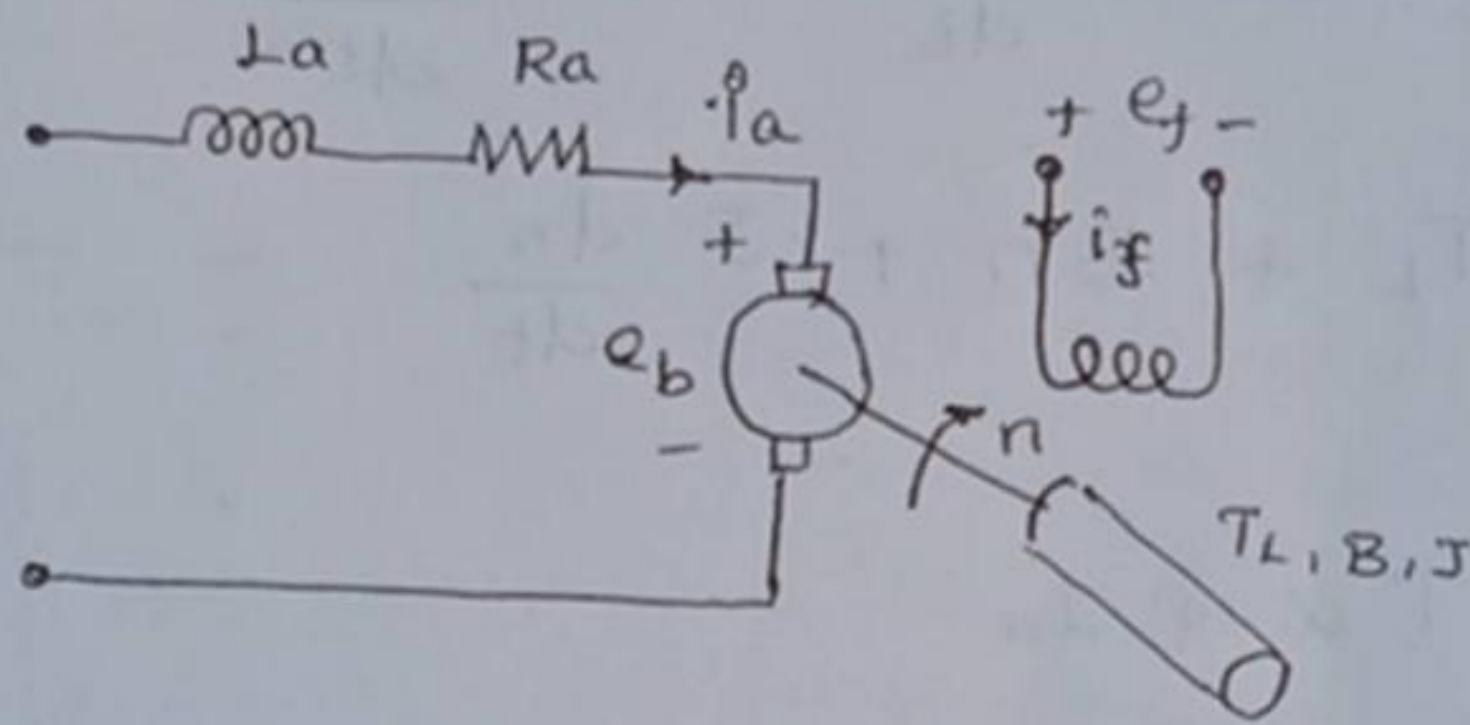
In braking operation, maximum torque per unit of the stator current is obtained when $\delta' = -90^\circ$.
When $\delta' = -90^\circ$, the stator current is reversed.
 δ' is the angle between rotating field produced by the stator and rotor.

UNIT-V

DESIGN OF CONTROLLER FOR DRIVES

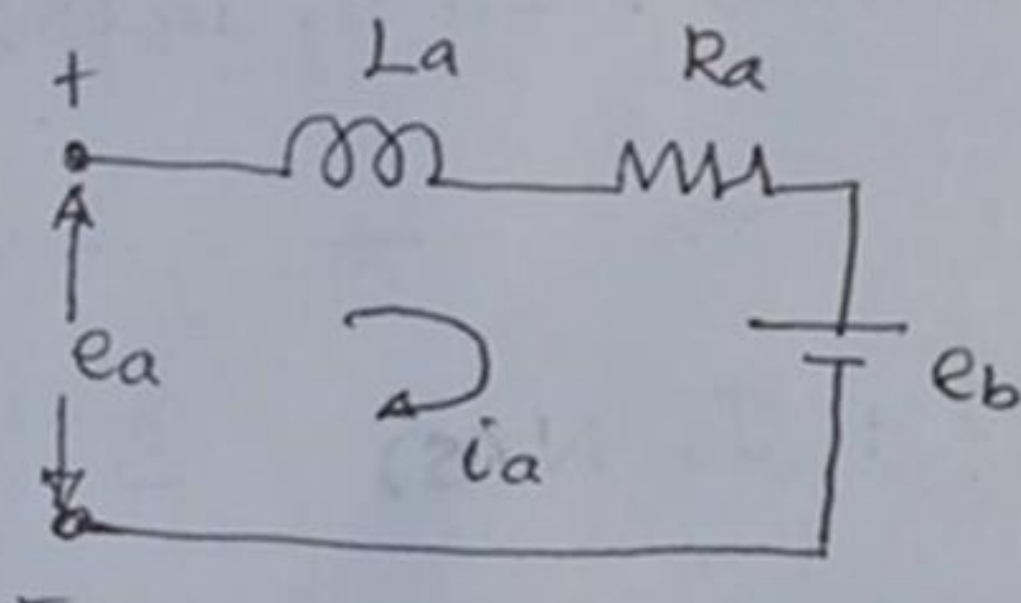
Transfer Function of Armature Control method of Separately Excited DC Motor and Load =

consider a separately excited dc motor, In armature voltage control method, field current is constant but armature voltage is varied.



We know that $N \propto V_s$, by using this method we can get below rated speed.

Electrical Analysis



Apply KVL

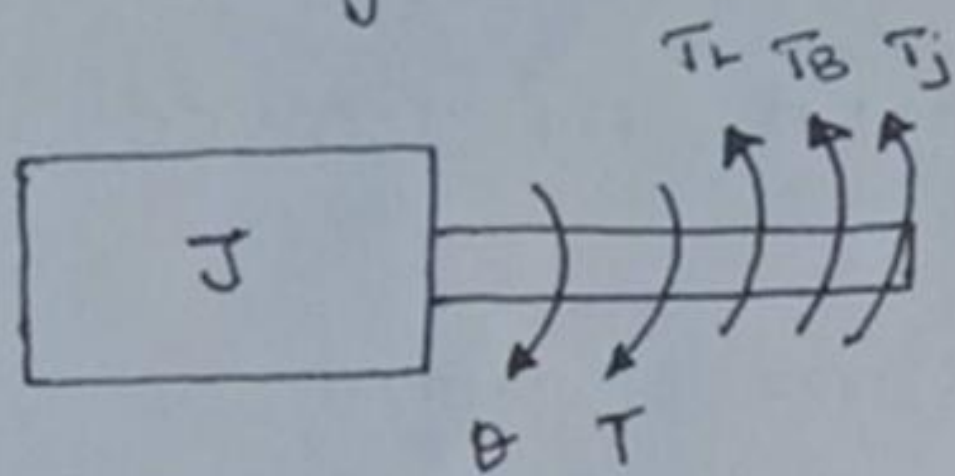
$$e_a = e_b + R_a i_a + L_a \frac{di_a}{dt} \quad \text{--- (1)}$$

where

$$e_b \propto \phi \cdot \frac{d\phi}{dt} \quad \left\{ \frac{d\phi}{dt} = \text{speed } (n) \right\}$$

$$e_b = k_a \phi n \quad \text{--- (2)}$$

Mechanical Analysis:-



Applying Newton's law, we can get torque balance equation.

$$T = T_L + T_B + T_J$$

$$T = T_L + B \frac{d\theta}{dt} + J \frac{d^2\theta}{dt^2}$$

$$T = T_L + Bn + J \frac{dn}{dt} \quad \text{--- --- --- (3)}$$

where

$$T \propto \phi I_a$$

$$T = K_a \phi I_a \quad \text{--- --- --- (4)}$$

Taking Laplace equation (1) to (4), we get

$$E_a(s) = E_b(s) + R_a I_a(s) + L_a s \cdot I_a(s) \quad \text{--- (5)}$$

$$E_b(s) = K_a \phi N(s) \quad \text{--- --- --- (6)}$$

$$T(s) = T_L(s) + B N(s) + J s N(s) \quad \text{--- --- --- (7)}$$

$$T(s) = K_a \phi I_a(s) \quad \text{--- --- --- (8)}$$

From equation (5),

$$E_a(s) = E_b(s) + R_a I_a(s) + L_a s \cdot I_a(s)$$

$$E_a(s) = E_b(s) + I_a(s) [R_a + s \cdot L_a]$$

$$I_a(s) [R_a + s \cdot L_a] = E_a(s) - E_b(s)$$

$$I_a(s) = \frac{E_a(s) - E_b(s)}{R_a [1 + \frac{sL_a}{R_a}]}$$

Finally

$$I_a(s) = \frac{1}{R_a} \frac{[E_a(s) - E_b(s)]}{1 + s\tau_a} \quad \text{--- (9)}$$

where $\tau = \frac{L_a}{R_a} \rightarrow$ electrical time constant of the armature circuit

From equation (7),

$$T(s) = T_L(s) + B N(s) + J_s N(s)$$

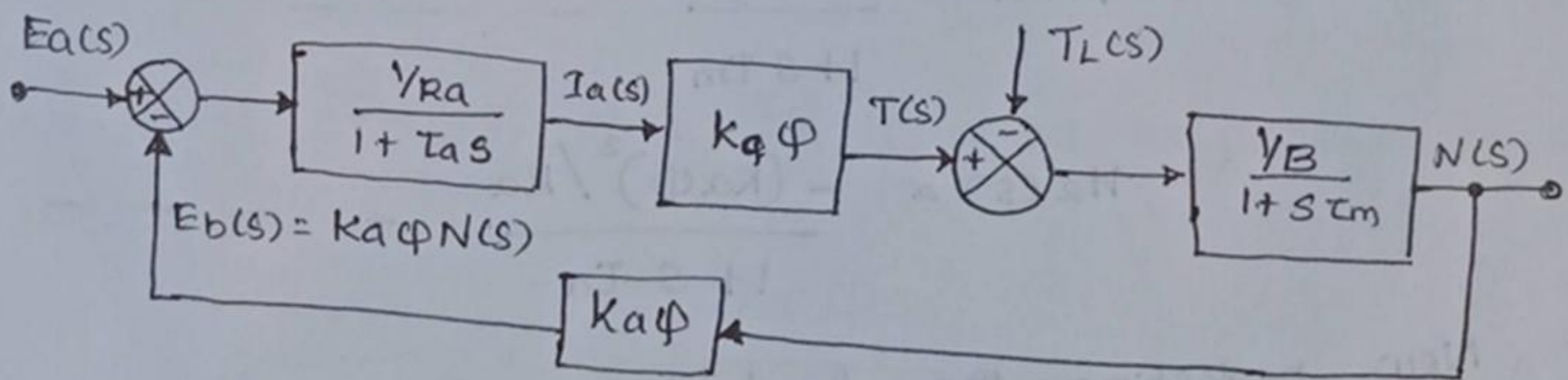
$$T(s) = T_L(s) + N(s) [B + J_s]$$

$$N(s) = \frac{T(s) - T_L(s)}{B + J_s}$$

$$N(s) = \frac{1}{B} \frac{[T(s) - T_L(s)]}{1 + T_m s} \quad \text{--- (10)}$$

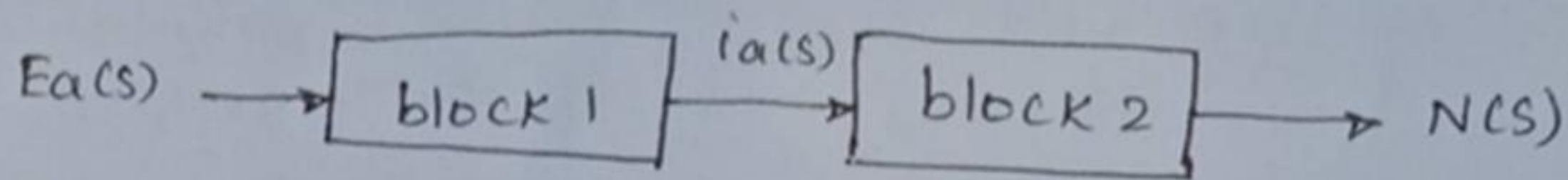
where $T_m \rightarrow \frac{J}{B} =$ mechanical time constant of the motor

These relationships are shown in block diagram



In this block diagram, the feedback loop present in the form of back emf. This back emf provides to moderate speed regulation inherent in the separately excited dc motor.

The block diagram can be obtained for the change in speed $N(s)$ due to disturbances in applied voltage $E_a(s)$ and load torque $T_L(s)$.



$N(s) = \text{block 1 transfer function} + \text{block 2 transfer function}$

$$\text{block 1} \Rightarrow \frac{G_1(s)}{1 + G_1(s) H_1(s)} \cdot E_a(s)$$

$$\text{block 2} \Rightarrow \frac{G_2(s)}{1 + G_2(s) H_2(s)} \cdot T_L(s)$$

$$\therefore N(s) = \frac{G_1(s)}{1 + G_1(s) H_1(s)} \cdot E_a(s) + \frac{G_2(s)}{1 + G_2(s) H_2(s)} \cdot T_L(s) \quad \text{--- (11)}$$

where,

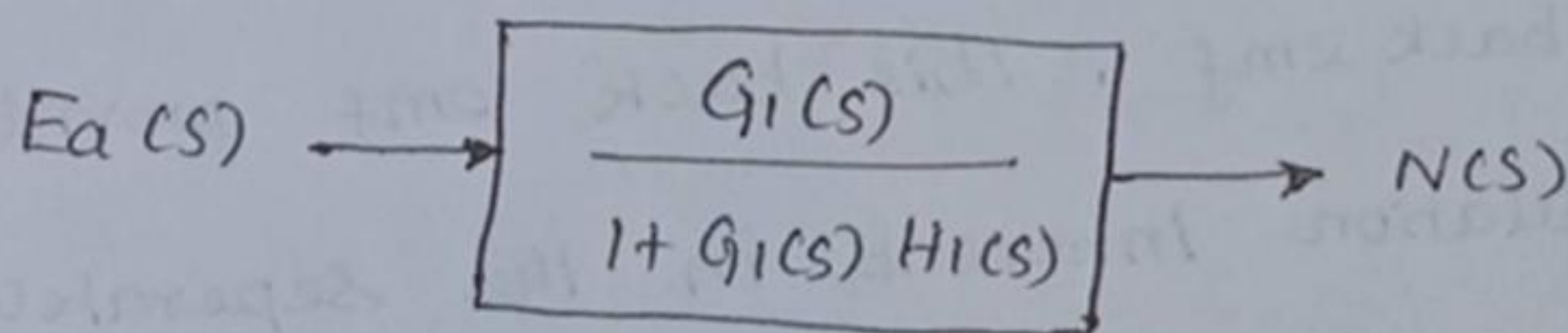
$$G_1(s) = \frac{Y_{Ra}}{1 + s T_a} (K_a \phi) \cdot \frac{Y_B}{1 + s T_m} \quad \text{--- (12)}$$

$$H_1(s) = K_a \phi \quad \text{--- (13)}$$

$$G_2(s) = \frac{-(Y_B)}{1 + s T_m} \quad \text{--- (14)}$$

$$H_2(s) = \frac{-(K_a \phi)^2 / R_a}{1 + s T_a} \quad \text{--- (15)}$$

Now neglecting the load torque T_L , the block diagram becomes



$$\frac{N(s)}{E_a(s)} = \frac{\frac{K_a \phi}{R_a \cdot B}}{(1+s\tau_a)(1+s\tau_m)}$$

$$= \frac{1 + \frac{(K_a \phi) / R_a \cdot B}{(1+s\tau_a)(1+s\tau_m)} \cdot K_a \phi}{\frac{(K_a \phi) / R_a \cdot B}{(1+s\tau_a)(1+s\tau_m) + (K_a \phi) / R_a \cdot B}}$$

$$\frac{N(s)}{E_a(s)} = \frac{(K_a \phi) / R_a \cdot B}{(1+s\tau_a)(1+s\tau_m) + (K_a \phi)^2 / R_a \cdot B}$$

$$= \frac{K_a \phi / R_a \cdot B}{R_a \cdot B (1+s\tau_a)(1+s\tau_m) + (K_a \phi)^2}$$

$$= \frac{K_a \phi}{R_a \cdot B}$$

$$\therefore \frac{N(s)}{E_a(s)} = \frac{K_a \cdot \phi}{R_a \cdot B (1+s\tau_a)(1+s\tau_m) + (K_a \phi)^2} \quad \text{--- (16)}$$

if $\tau_a \ll \tau_m$, then τ_a can be neglected in eqn (16)

we get

$$\frac{N(s)}{E_a(s)} = \frac{K_a \phi}{R_a \cdot B (1+s\tau_m) + (K_a \phi)^2}$$

$$= \frac{K_a \phi}{R_a \cdot B + R_a \cdot B \cdot s\tau_m + (K_a \phi)^2}$$

$\therefore (R_a \cdot B + (K_a \phi)^2)$ in all terms.

$$\frac{N(s)}{E_a(s)} = \frac{k_a \phi / R_a \cdot B + (k_a \phi)^2}{1 + \frac{s \cdot R_a \cdot B}{R_a \cdot B + (k_a \phi)^2} \cdot \tau_m}$$

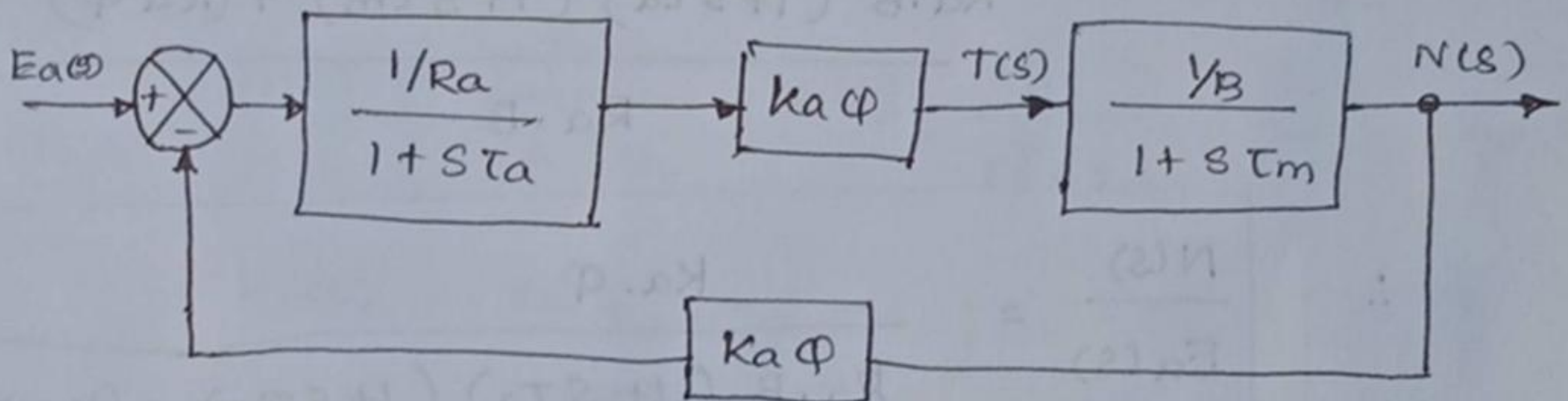
$$\therefore \boxed{\frac{N(s)}{E_a(s)} = \frac{K_m}{1 + s \tau_{m1}}} \quad \text{--- (17)}$$

where $K_m = \frac{k_a \phi}{(k_a \phi)^2 + R_a \cdot B}$

$$\tau_{m1} = \frac{R_a \cdot B \cdot \tau_m}{R_a \cdot B + (k_a \phi)^2}$$

$$\tau_{m1} < \tau_m$$

Neglecting τ_L , then block diagram becomes



So $\frac{N(s)}{I_a(s)} = \frac{1/B}{1 + s \tau_m} k_a \phi$

$$= \frac{k_a \phi / B}{1 + s \tau_m}$$

$$\frac{N(s)}{I_a(s)} = \frac{K_{m2}}{1 + s \tau_m} \quad \text{--- (18)}$$

where $K_{m2} = \frac{k_a \phi}{B}$

$$\frac{I_a(s)}{E_a(s)} = \frac{N(s)}{E_a(s)} \times \frac{I_a(s)}{N(s)}$$

$$= \frac{k_m}{1 + sT_{m1}} \times \frac{(1 + sT_m) B}{k_a \phi}$$

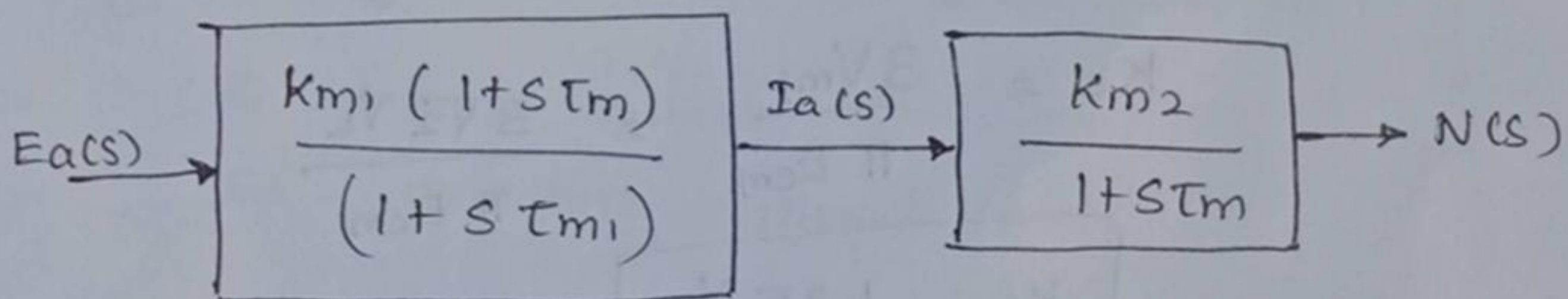
$$= \frac{k_m \cdot B (1 + sT_m)}{k_a \phi (1 + sT_{m1})}$$

$$\frac{I_a(s)}{E_a(s)} = \frac{k_{m1} (1 + sT_m)}{(1 + sT_{m1})}$$

where $k_{m1} = \frac{k_m \cdot B}{k_a \phi}$

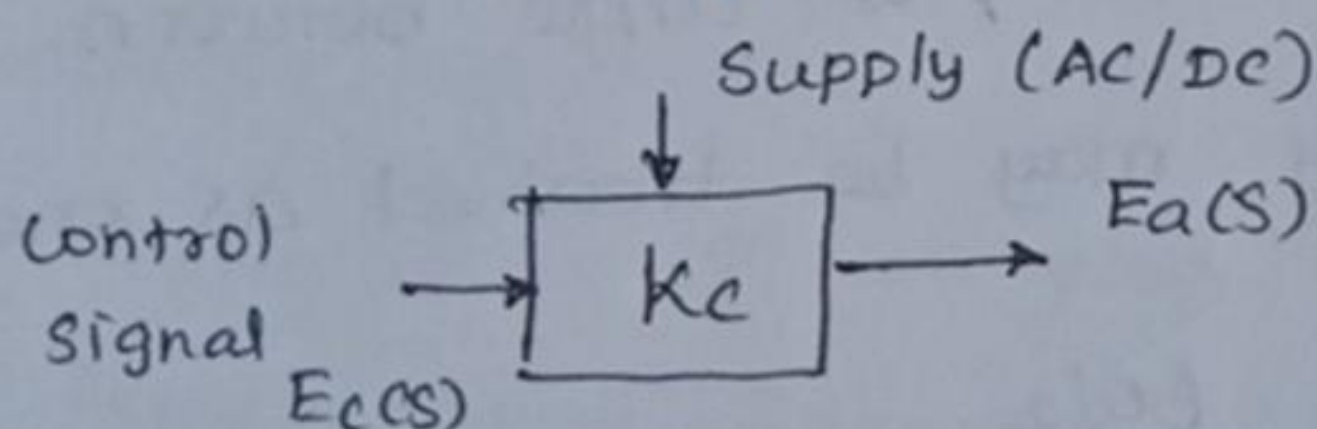
$$k_m = k_{m1} \times k_{m2}$$

Thus the motor can be represented, for the purpose of analysing if for voltage control,



Transfer function of power converter:

The simple block diagram of power converter gain k_e with input, output and control signal



converter transfer function is represented by

$$G_c(s) = \frac{E_a(s)}{E_c(s)}$$
$$= \frac{K_c}{1 + sT_c}$$

$K_c \rightarrow$ converter gain, $T_c \rightarrow$ converter time delay

Converter gain for a maximum control voltage E_{cm} is given by

$$K_c = \frac{2V_m}{\pi E_{cm}}$$
$$= \frac{2\sqrt{2}V_s}{\pi E_{cm}}$$

$$K_c = 0.9 \frac{V_s}{E_{cm}} \quad \text{--- --- --- --- --- } \textcircled{1}$$

For 3 ϕ converter, the converter gain is given by

$$K_c = \frac{3V_{mf}}{\pi E_{cm}} = \frac{3\sqrt{2}V_L}{\pi E_{cm}}$$

$$K_c = \frac{1.35 V_L}{E_{cm}}$$

Only a thyristor is switched on, its triggering angle cannot be changed. The new triggering delay can be implemented within 60° , i.e. angle between two thyristor gating. The delay may be treated as one half of this interval,

$$T_c = \frac{60/2}{360} \times \text{time period of one cycle.}$$

$$T_c = \frac{1}{12} \times \frac{1}{f_s} \text{ sec.}$$

then

$$G_c(s) = K_c \cdot e^{-sT_c}$$

The above eqn can be approximated as first order time lag as,

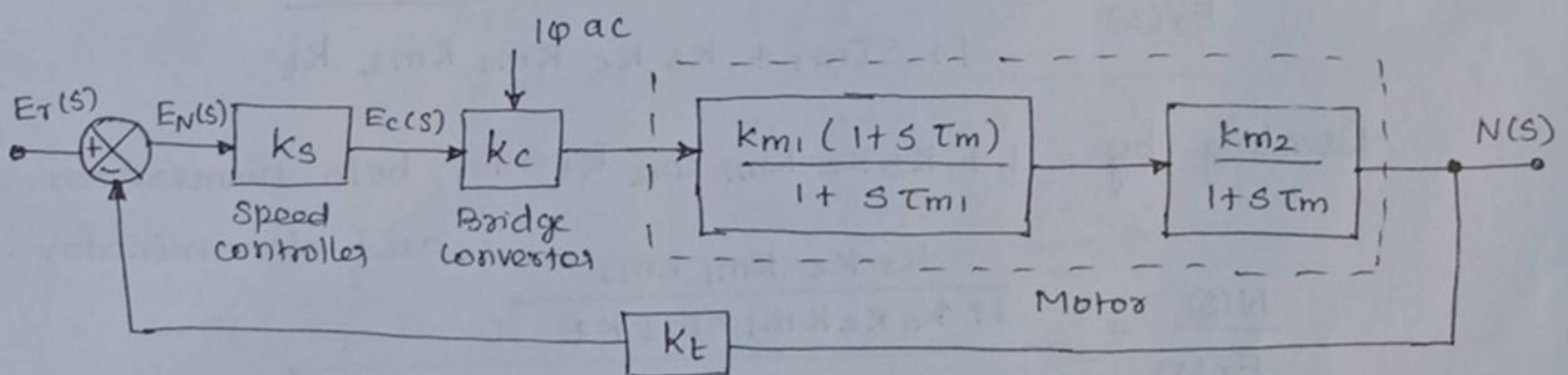
$$G_c(s) = \frac{K_c}{1 + sT_c}$$

Speed controller:-

If a DC generator is attached to the motor shaft, a speed signal can be feedback and the error $E_N(s)$ used to control the armature voltage. The applied armature voltage is controlled by a single phase or three phase full converter or a DC chopper.

E_c - control voltage

E_a - armature voltage.



Most commonly used speed controllers are,

- (i) Proportional (P) controller
- (ii) Proportional Integral (PI) controller.

Proportional Controller

The transfer function of the speed controller is

$$\frac{N(s)}{E_r(s)} = \frac{G(s)}{1 + G(s)H(s)} \quad \text{--- --- --- --- --- } \textcircled{1}$$

where

$$G(s) = K_s \cdot K_c \cdot \frac{K_{m1}(1 + sT_{m1})}{1 + sT_{m1}} \times \frac{K_{m2}}{1 + sT_{m1}}$$

$$G(s) = \frac{K_s \cdot K_c \cdot K_{m1} \cdot K_{m2}}{1 + sT_{m1}} \quad \text{--- --- --- --- --- } \textcircled{2}$$

$$H(s) = K_t \quad \text{--- --- --- --- --- } \textcircled{3}$$

Sub eqn ② & ③ in eqn ①, we get

$$\begin{aligned} \frac{N(s)}{E_r(s)} &= \frac{K_s K_c K_{m1} K_{m2} / (1 + sT_{m1})}{1 + \frac{K_s K_c K_{m1} K_{m2}}{1 + sT_{m1}} \times K_t} \\ &= \frac{K_s K_c K_{m1} K_{m2} / (1 + sT_{m1})}{1 + sT_{m1} + K_s K_c K_{m1} K_{m2} K_t / (1 + sT_{m1})} \end{aligned}$$

$$\frac{N(s)}{E_r(s)} = \frac{K_s K_c K_{m1} K_{m2}}{1 + sT_{m1} + K_s K_c K_{m1} K_{m2} K_t}$$

Dividing by $1 + K_s K_c K_{m1} K_{m2} K_t$ in both numerator and denominator.

$$\frac{N(s)}{E_r(s)} = \frac{\frac{K_s K_c K_{m1} K_{m2}}{1 + K_s K_c K_{m1} K_{m2} K_t}}{1 + \frac{sT_{m1}}{1 + K_s K_c K_{m1} K_{m2} K_t}}$$

$$\boxed{\frac{N(s)}{E_r(s)} = \frac{K_i}{1 + sT_i}} \quad \text{--- --- --- --- --- } \textcircled{4}$$

where $K_1 = \frac{K_s K_c K_{m1} K_{m2}}{1 + K_s K_c K_{m1} K_{m2} K_t}$ ----- (5)

$$\tau_1 = \frac{\tau_{m1}}{1 + K_s K_c K_{m1} K_{m2} K_t}$$
 ----- (6)

If $K_s K_c K_{m1} K_{m2} K_t \gg 1$, then neglecting the term 1 in the denominator and we can get,

$$K_1 = \frac{K_s K_c K_{m1} K_{m2}}{K_s K_c K_{m1} K_{m2} K_t}$$

$$\therefore \boxed{K_1 = \frac{1}{K_t}}$$
 ----- (7)

$$\tau_1 = \frac{\tau_{m1}}{K_s K_c K_{m1} K_{m2} K_t}$$
 ----- (8)

W.K.T

$$\frac{I_a(s)}{N(s)} = \frac{1 + s\tau_m}{K_{m2}}$$

then $\frac{I_a(s)}{E_r(s)} = \frac{N(s)}{E_r(s)} \times \frac{I_a(s)}{N(s)}$ ----- (9)

$$\frac{I_a(s)}{E_r(s)} = \frac{K_1}{1 + s\tau_1} \times \frac{1 + s\tau_m}{K_{m2}}$$

$$\boxed{I_a(s) = \frac{K_1 (1 + s\tau_m)}{K_{m2} (1 + s\tau_1)} \times E_r(s)}$$
 ----- (10)

If the input is a step input, ie

$$E_r(s) = \frac{E_r}{s}$$

$$\therefore I_a(s) = \frac{K_1 (1 + s\tau_m)}{K_{m2} (1 + s\tau_1)} \times \frac{E_r}{s}$$
 ----- (11)

$$I_a(s) = \frac{K_1 E_r (1 + sT_m)}{K_{m2} s \tau_1 \left[s + \frac{1}{\tau_1} \right]}$$

Dividing by $K_{m2} \tau_1$

$$= \frac{K_1 E_r (1 + sT_m) / K_{m2} \cdot \tau_1}{\frac{K_{m2} s \tau_1}{K_{m2} \tau_1} \left[s + \frac{1}{\tau_1} \right]}$$

$$I_a(s) = \frac{K_1 E_r [1 + sT_m] / K_{m2} \tau_1}{s \left[s + \frac{1}{\tau_1} \right]}$$

Using partial fraction method,

$$\frac{K_1 E_r [1 + sT_m] / K_{m2} \tau_1}{s \left[s + \frac{1}{\tau_1} \right]} = \frac{A_1}{s} + \frac{A_2}{s + \frac{1}{\tau_1}}$$

$$= \frac{A_1 \left[s + \frac{1}{\tau_1} \right] + A_2 s}{s \left[s + \frac{1}{\tau_1} \right]}$$

$$\frac{K_1 E_r [1 + sT_m]}{K_{m2} \tau_1} = A_1 \left[s + \frac{1}{\tau_1} \right] + A_2 (s)$$

$$\frac{K_1 E_r}{K_{m2} \tau_1} + \frac{K_1 E_r T_m}{K_{m2} \tau_1} \cdot s = s [A_1 + A_2] + \frac{A_1}{\tau_1}$$

Equating constant term on both sides,

$$\frac{K_1 E_r}{K_{m2} \tau_1} = \frac{A_1}{\tau_1}$$

$$\boxed{A_1 = \frac{K_1 E_r}{K_{m2}}}$$

(12)

Equating 's' terms on both sides,

$$\frac{K_1 E_r \tau_m}{K m_2 \tau_1} = A_1 + A_2$$

$$\frac{K_1 E_r \tau_m}{K m_2 \tau_1} = \frac{K_1 E_r}{K m_2} + A_2$$

$$A_2 = \frac{K_1 E_r}{K m_2} \left[\frac{\tau_m}{\tau_1} - 1 \right] \dots \dots \dots (13)$$

Then,

$$I_a(s) = \frac{K_1 E_r}{K m_2 s} + \left[\frac{K_1 E_r}{K m_2} \left[\frac{\tau_m}{\tau_1} - 1 \right] \right] / \left[s + \frac{1}{\tau_1} \right]$$

$$I_a(s) = \frac{K_1 E_r}{K m_2 s} + \frac{E_r K_1 (\tau_m - \tau_1)}{K m_2 \tau_1 \left(s + \frac{1}{\tau_1} \right)}$$

$$I_a(s) = \frac{E_r K_1}{K m_2} \left[\frac{1}{s} + \frac{\tau_m - \tau_1}{\tau_1} \times \frac{1}{\left(s + \frac{1}{\tau_1} \right)} \right] \dots \dots \dots (14)$$

Taking inverse Laplace transform,

$$I_a(t) = \frac{K_1 E_r}{K m_2} \left[1 + \frac{\tau_m - \tau_1}{\tau_1} \cdot e^{-t/\tau_1} \right] \dots \dots \dots (15)$$

Since $\tau_m \gg \tau_1$, τ_1 can be neglected,

when $t \rightarrow \infty$ $I_a(t) \rightarrow I_a(\infty)$

$$\therefore I_a(\infty) = \frac{E_r K_1}{K m_2} [1 + e^{-\infty}]$$

$$\boxed{I_a(\infty) = \frac{E_r K_1}{K m_2}} \dots \dots \dots (16)$$

Normalizing the current $I_a(t)$ with respect to the steady state current $I_a(\infty)$,

$$\frac{I_a(t)}{I_a(\alpha)} = \frac{\frac{E_r k_1}{K_{m2}} \cdot \left[1 + \frac{\tau_{m1} - \tau_1}{\tau_1} \cdot e^{-t/\tau_1} \right]}{\frac{E_r k_1}{K_{m2}}}$$

$$\boxed{\frac{I_a(t)}{I_a(\alpha)} = 1 + \frac{\tau_{m1} - \tau_1}{\tau_1} \cdot e^{-t/\tau_1}} \quad \text{--- --- (7)}$$

The small change in the input E_r will result in a large sudden change in current that decays slowly. This transient over current is undesirable as the point of converter rating and protection. This is particularly the case of starting or other sudden changes. For this purpose current controller is used.

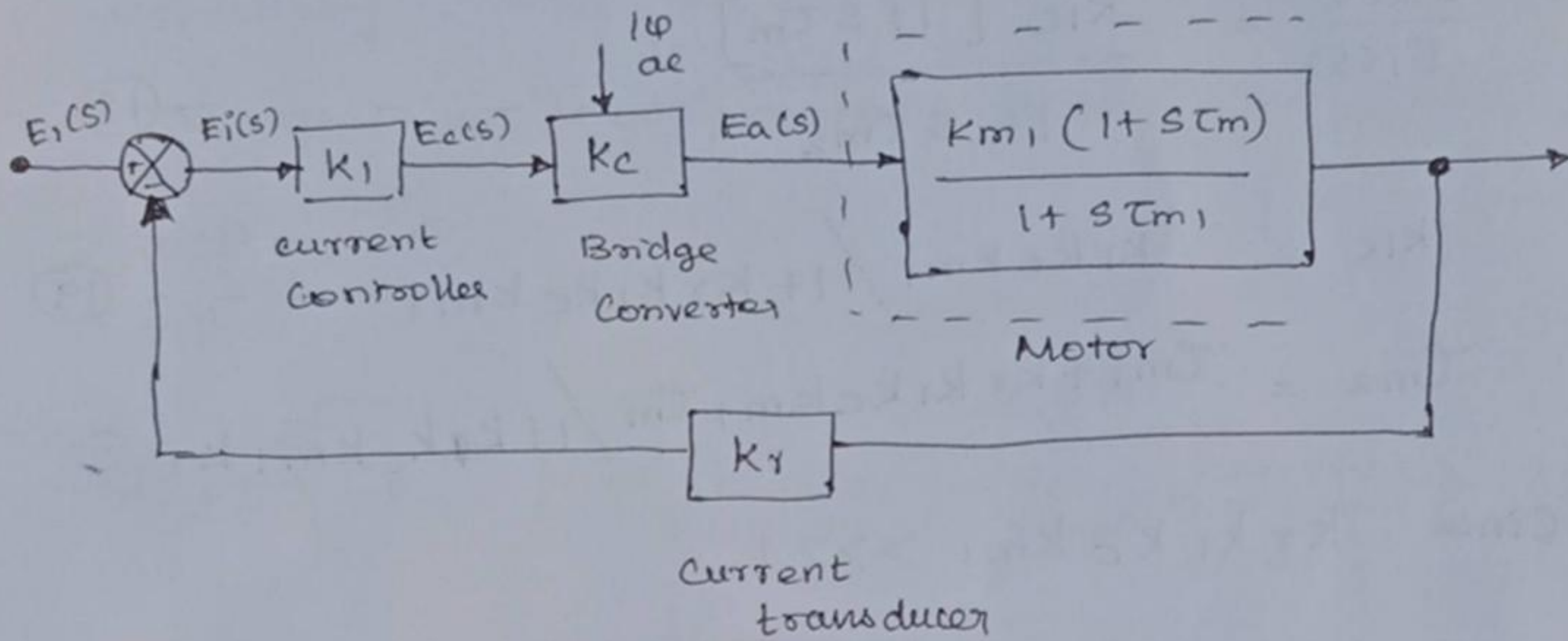
Current controller:-

If armature losses are neglected, climbing the speed error will limit the speed but not the current. However, a current limit can be implemented if we construct an inner current control loop using the clamped speed error as the current reference. Both P controller and PI controller for the current control loop are discussed,

P - Controller:-

Inner current control loop has K_r is gain of the current transducer, which may be

a sampling resistor in the armature circuit. The gain of the current controller is K_i , which are assumed to be a proportional controller.



In the current control loop, controlled output is $I_a(s)$ and the input is the speed error as the current reference. The transfer function is,

$$\begin{aligned} \frac{I_a(s)}{E_i(s)} &= \frac{K_i K_c K_{m1} (1+sT_m)}{(1+sT_m) [1 + K_i K_c K_{m1} (1+sT_m) K_r]} \\ &= \frac{K_i K_c K_{m1} (1+sT_m) / (1+sT_m)}{1 + sT_m + K_i K_c K_{m1} (1+sT_m) K_r / (1+sT_m)} \\ &= \frac{K_i K_c K_{m1} + K_i K_c K_{m1} T_m s}{1 + sT_m + K_i K_c K_{m1} K_r + K_i K_c K_{m1} K_r T_m s} \\ \frac{I_a(s)}{E_i(s)} &= \frac{K_i K_c K_{m1} + K_i K_c K_{m1} T_m s}{1 + K_i K_c K_{m1} K_r + s [T_m + K_i K_c K_{m1} K_r T_m]} \end{aligned}$$

Dividing by $1 + K_i K_c K_{m1} K_r$ in numerator & denominator,

$$\frac{I_a(s)}{E_1(s)} = \frac{k_1 k_c k_{m1} (1 + s \tau_m) / (1 + k_1 k_c k_{m1} k_r)}{1 + s \left[\frac{\tau_{m1} + k_1 k_c k_{m1} k_r \tau_m}{1 + k_1 k_c k_{m1} k_r} \right]}$$

$$\frac{I_a(s)}{E_1(s)} = \frac{k_{1c} [1 + s \tau_m]}{1 + s \tau_{m2}} \quad \text{--- (18)}$$

$$k_{1c} = k_1 k_c k_{m1} / (1 + k_r k_1 k_c k_{m1}) \quad \text{--- (19)}$$

$$\tau_{m2} = \tau_{m1} + k_r k_1 k_c k_{m1} \tau_m / (1 + k_r k_1 k_c k_{m1} k_r) \quad \text{--- (20)}$$

Since $k_r k_1 k_c k_{m1} \gg 1$

$$k_{1c} = \frac{k_1 k_c k_{m1}}{k_1 k_c k_{m1} k_r} = \frac{1}{k_r} \quad \text{--- (21)}$$

$$\begin{aligned} \tau_{m2} &= \frac{\tau_{m1} + k_1 k_c k_{m1} k_r \tau_m}{k_1 k_c k_{m1} k_r} \\ &= \frac{\tau_{m1}}{k_1 k_c k_{m1} k_r} + \tau_m \quad \text{--- (22)} \end{aligned}$$

Also $\tau_m \gg \tau_{m1}$, $\therefore \tau_{m1}$ is neglected,

$$\boxed{\tau_{m2} = \tau_m} \quad \text{--- (23)}$$

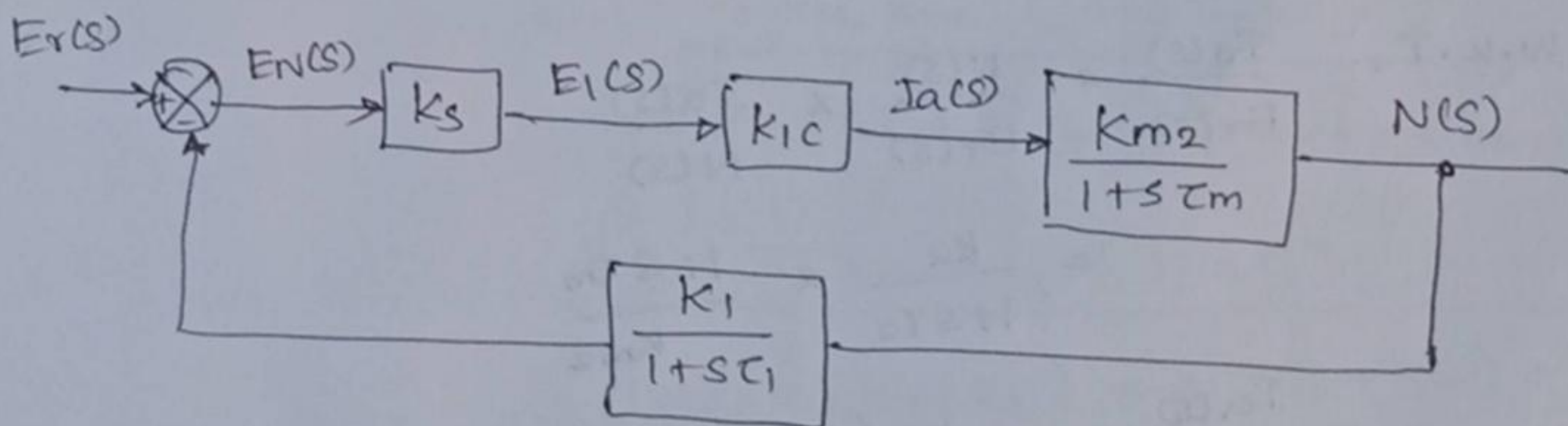
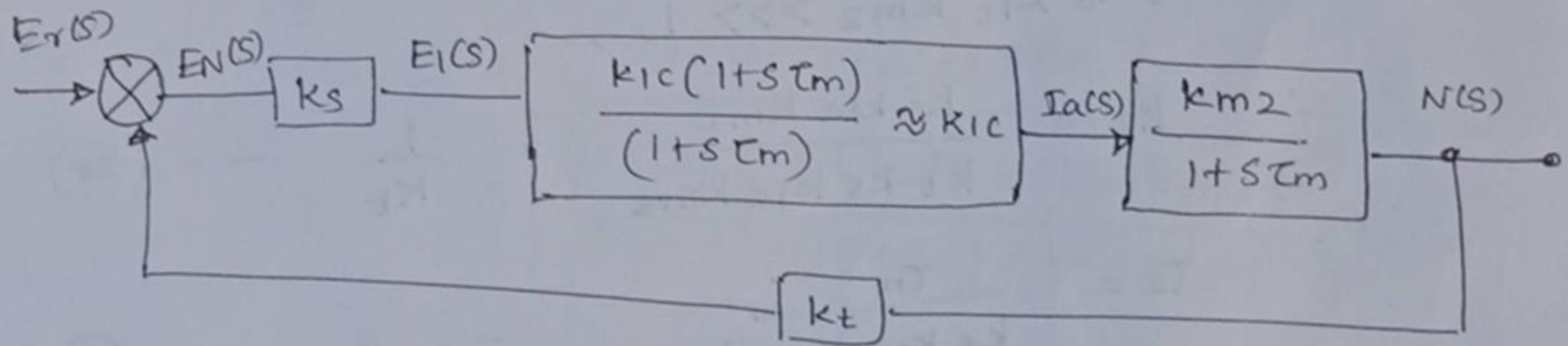
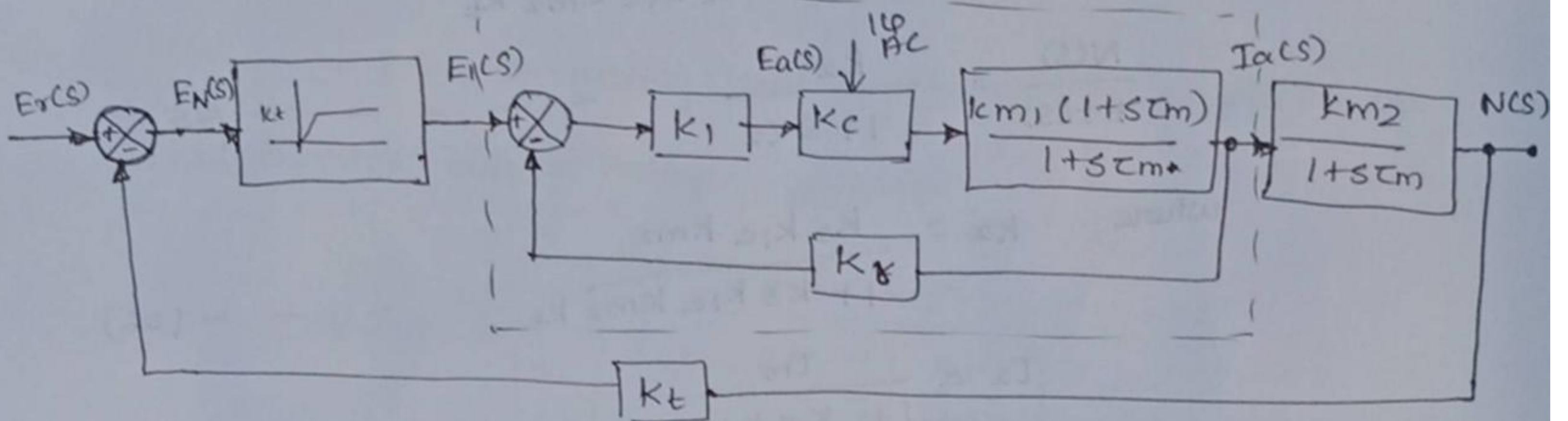
A pole zero cancellation is possible for the above equation, resulting no overshoot or time delay due to armature circuit electrical time constant and the converter delay.

$$\therefore \frac{I_a(s)}{E_1(s)} = \frac{k_{1c} (1 + s \tau_m)}{1 + s \tau_{m2}}$$

$$k_{1c} = \frac{1}{k_r}, \text{ then} \quad \text{--- (24)}$$

$$\frac{I_a(s)}{E_1(s)} = \frac{1 + sT_m}{K_r (1 + sT_{m2})}$$

I_a is directly related to E_1 , to limit on E_1 will effectively limit the current. The inner current loop can now be incorporated within the speed control loop.



Without Techogenerator:-

$$\frac{N(s)}{E_r(s)} = \frac{K_s k_{i2} k_{m2} \times \frac{1}{1+sT_m}}{1 + \frac{K_s k_{i2} k_{m2} k_t}{1+sT_m}}$$

$$= \frac{K_s k_{i2} k_{m2}}{1 + sT_m + K_s k_{i2} k_{m2} k_t}$$

$$\frac{N(s)}{E_r(s)} = \frac{k_s k_{ic} k_{m2}}{[1 + k_s k_{ic} k_{m2} k_t] + s T_m}$$

Dividing by $1 + k_s k_{ic} k_{m2} k_t$ in numerator & denominator

$$\frac{N(s)}{E_r(s)} = \frac{\frac{k_s k_{ic} k_{m2}}{1 + k_s k_{ic} k_{m2} k_t}}{1 + \frac{s T_m}{1 + k_s k_{ic} k_{m2} k_t}}$$

$$\frac{N(s)}{E_r(s)} = \frac{k_2}{1 + s T_2} \quad \text{--- (25)}$$

where $k_2 = \frac{k_s k_{ic} k_{m2}}{1 + k_s k_{ic} k_{m2} k_t}$ --- (26)

$$T_2 = \frac{T_m}{1 + k_s k_{ic} k_{m2} k_t}$$
 --- (27)

Since $k_t k_s k_{ic} k_{m2} \gg \gg 1$,

$$\therefore k_2 = \frac{k_s k_{ic} k_{m2}}{k_t k_s k_{ic} k_{m2}} = \frac{1}{k_t}$$
 --- (28)

$$T_2 = \frac{T_m}{k_s k_{ic} k_{m2} k_t}$$
 --- (29)

W.K.T, $\frac{I_a(s)}{E_r(s)} = \frac{N(s)}{E_r(s)} \times \frac{I_a(s)}{N(s)}$

$$= \frac{k_2}{1 + s T_2} \times \frac{1 + s T_m}{k_{m2}}$$

$$\frac{I_a(s)}{E_r(s)} = \frac{k_2 (1 + s T_m)}{k_{m2} (1 + s T_2)}$$
 --- (30)

If during load changes, speed error is large, $E_1 = \hat{E}_1$ (max) current is limited to maximum value

$$I_a = k_{ic} \hat{E}_1$$

the speed 'N' is given by

$$N(s) = I(s) \cdot \frac{K_{m2}}{1+s\tau_m} \quad \text{--- (31)}$$

For step input, $N(s) = \frac{I_a}{s} \cdot \frac{K_{m2}}{1+s\tau_m} \quad \text{--- (32)}$

Here motor speed is directly proportional to I_a .

Tachogenerator with filter:-

Filter is mainly used to reduce the ripple in the tachogenerator output voltage.

$$\begin{aligned} \frac{N(s)}{E_r(s)} &= \frac{K_s K_{ic} K_{m2} / 1+s\tau_m}{1 + \left[\frac{K_s K_{ic} K_{m2}}{1+s\tau_m} \right] \left[\frac{K_t}{1+s\tau_t} \right]} \\ &= \frac{K_s K_{ic} K_{m2} / 1+s\tau_m}{\left[\frac{(1+s\tau_m)(1+s\tau_t) + K_s K_{ic} K_{m2} K_t}{(1+s\tau_m)(1+s\tau_t)} \right]} \\ &= \frac{K_s K_{ic} K_{m2} (1+s\tau_t)}{1+s\tau_t + s\tau_m + s^2\tau_m\tau_t + K_s K_{ic} K_{m2} K_t} \\ &= \frac{K_s K_{ic} K_{m2} (1+s\tau_t)}{(1 + K_s K_{ic} K_{m2} K_t) + s(\tau_t + \tau_m) + s^2\tau_m\tau_t} \end{aligned}$$

Dividing by $1 + K_t K_s K_{m2} K_{ic}$ in numerator and denominator.

$$\frac{N(s)}{E_r(s)} = \frac{\frac{K_s K_{m2} K_{ic} (1+s\tau_t)}{1 + K_t K_s K_{m2} K_{ic}}}{1 + \frac{s(\tau_t + \tau_m)}{1 + K_t K_s K_{m2} K_{ic}} + \frac{s^2\tau_m\tau_t}{1 + K_t K_s K_{m2} K_{ic}}}$$

$$\frac{N(s)}{E_r(s)} = \frac{k_s k_{m2} k_{ic} [1 + s\tau_t]}{\left[1 + \frac{s(\tau_t + \tau_m)}{k'} + \frac{s^2 \tau_m \tau_t}{k'} \right] [1 + k_t k_s k_{m2} k_{ic}]}$$

$$k' = 1 + k_t k_s k_{m2} k_{ic}$$

$\tau_t \rightarrow$ tachogenerator filter time constant

$$k_t k_s k_{m2} k_{ic} \gg \gg 1$$

$$k' = k_t k_s k_{m2} k_{ic}$$

We know that

$$\frac{I_a(s)}{E_r(s)} = \frac{N(s)}{E_r(s)} \times \frac{I_a(s)}{N(s)}$$

$$\frac{I_a(s)}{N(s)} = \frac{1 + s\tau_m}{k_{m2}}$$

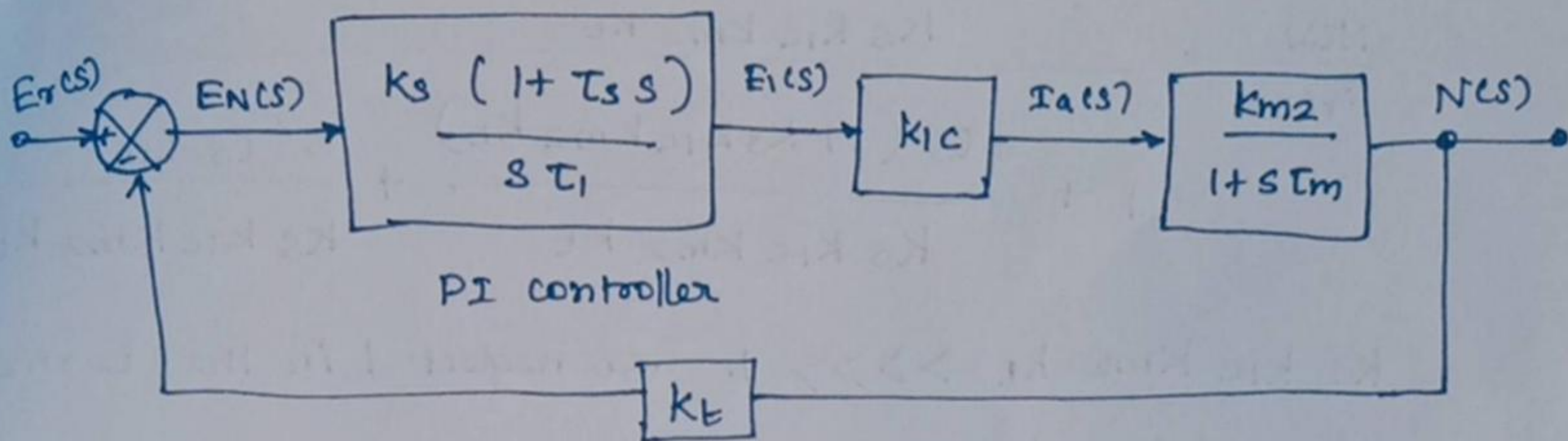
$$\frac{I_a(s)}{E_r(s)} = \frac{k_s k_{m2} k_{ic} (1 + s\tau_t)}{\left[1 + \frac{s(\tau_t + \tau_m)}{k'} + \frac{s^2 \tau_m \tau_t}{k'} \right] [1 + k_t k_s k_{m2} k_{ic}]} \times \frac{1 + s\tau_m}{k_{m2}}$$

$$= \frac{k_s k_{ic} [1 + s\tau_t] [1 + s\tau_m]}{[1 + k_t k_s k_{m2} k_{ic}] \left[1 + \frac{s[\tau_t + \tau_m]}{k'} + \frac{s^2 \tau_m \tau_t}{k'} \right]} \quad (34)$$

Proportional - Integral (PI) controller

- used to eliminate the steady state error and reduce the forward gain required.

To obtain this integral component the proportional speed controller is replaced by a proportional integral type controller.



Transfer function of P-controller = K_s

Transfer function of PI controller = $\frac{K_s [1 + s T_s]}{s T_s}$

$$G(s) = \frac{K_s K_i c K_m 2 [1 + T_s s]}{s T_s [1 + s T_m]}$$

$$H(s) = K_t$$

Transfer function, $\frac{N(s)}{E_r(s)} = \frac{G(s)}{1 + G(s)H(s)}$

$$\frac{N(s)}{E_r(s)} = \frac{K_s K_i c K_m 2 (1 + s T_s) / s T_s (1 + s T_m)}{1 + K_s K_i c K_m 2 K_t (1 + s T_s) / s T_s (1 + s T_m)}$$

$$= \frac{K_s K_i c K_m 2 (1 + s T_s) / s T_s (1 + s T_m)}{s T_s (1 + s T_m) + K_s K_i c K_m 2 K_t (1 + s T_s)}$$

$$= \frac{K_s K_i c K_m 2 [1 + s T_s]}{s T_s + s^2 T_s T_m + K_s K_i c K_m 2 K_t + K_s K_i c K_m 2 K_t T_s s}$$

$$= \frac{K_s K_i c K_m 2 (1 + s T_s)}{K_s K_i c K_m 2 K_t + s T_s [1 + K_s K_i c K_m 2 K_t] + s^2 T_s T_m}$$

Dividing by $K_s K_i c K_m 2 K_t$ in numerator and denominator.

$$\frac{N(s)}{E_r(s)} = \frac{k_s k_{ic} k_{m2} [1 + s\tau_s]}{k_s k_{ic} k_{m2} k_t} \left[1 + \frac{s\tau_s (1 + k_s k_{ic} k_{m2} k_t)}{k_s k_{ic} k_{m2} k_t} + \frac{s^2 \tau_s \tau_m}{k_s k_{ic} k_{m2} k_t} \right]$$

$k_s k_{ic} k_{m2} k_t \gg \gg \gg 1$. So neglect 1 in the term $1 + k_s k_{ic} k_{m2} k_t$, we get

$$\frac{N(s)}{E_r(s)} = \frac{\frac{1}{k_t} (1 + s\tau_s)}{1 + \frac{s\tau_s k_s k_{ic} k_{m2} k_t}{k_s k_{ic} k_{m2} k_t} + \frac{s^2 \tau_s \tau_m}{k_s k_{ic} k_{m2} k_t}}$$

Substitute $\tau_2 = \frac{\tau_m}{k_s k_{ic} k_{m2}}$, we get

$$\frac{N(s)}{E_r(s)} = \frac{\frac{1}{k_t} (1 + s\tau_s)}{1 + s\tau_s + s^2 \tau_s \tau_2}$$

W.K.T, $\frac{I_a(s)}{E_r(s)} = \frac{N(s)}{E_r(s)} \times \frac{I_a(s)}{N(s)}$

$$\frac{I_a(s)}{E_r(s)} = \frac{1 + s\tau_m}{k_{m2}}$$

$$\frac{I_a(s)}{E_r(s)} = \frac{1}{k_t} \cdot \frac{1 + s\tau_s}{1 + s\tau_s + s^2 \tau_s \tau_2} \times \frac{1 + s\tau_m}{k_{m2}}$$

$$\boxed{\frac{I_a(s)}{E_r(s)} = \frac{1}{k_t k_{m2}} \cdot \frac{(1 + s\tau_s)(1 + s\tau_m)}{1 + s\tau_s + s^2 \tau_s \tau_2}}$$